The spectrum of composition operators induced by a rotation in the space of all analytic functions on the disc

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Abstract

A characterization of those points of the unit circle which belong to the spectrum of a composition operator C_φ, defined by a rotation $\varphi(z) = rz$ with $|r| = 1$, on the space $H_0(\mathbb{D})$ of all analytic functions on the unit disc which vanish at 0, is given. Examples show that the spectrum of C_φ need not be closed. In these examples the spectrum is dense but point 1 may or may not belong to it, and this is related to Diophantine approximation.

The aim of this note is to present a few results about the spectrum $\sigma(C_{\varphi_r}, H_0(\mathbb{D}))$ of composition operators, defined by a self map of the unit disc $\mathbb{D}$ given by $\varphi_r(z) := rz$, for a complex number r with $|r| = 1$, when acting on the space $H_0(\mathbb{D})$ of all analytic functions on the disc $\mathbb{D}$ vanishing at 0. Our results complement recent work by Arendt, Celariès and Chalendar [2]. The main result Theorem 2 provides a characterization of the points of the unit circle which belong to the spectrum of a composition operator C_{φ_r}. Moreover, as a consequence of Theorem 4, we show that there exist $r, s \in \mathbb{C}$ with $|r| = |s| = 1$, which are not roots of unity, such that the corresponding self-maps φ_r and φ_s satisfy $1 \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$, in which case $\sigma(C_{\varphi_r}, H_0(\mathbb{D}))$ is not closed, and $1 \in \sigma(C_{\varphi_s}, H_0(\mathbb{D}))$. The example of r uses Diophantine irrational numbers, see Theorem 4 (1); whereas the example of s as indicated requires a construction given in Lemma 3 of irrational numbers x with a sequence of rational numbers very rapidly converging to x and an argument given in Theorem 4 (2).

Here the space $H(\mathbb{D})$ of all analytic functions on $\mathbb{D}$ is endowed with the complete metrizable locally convex topology of uniform convergence on the compact subsets of $\mathbb{D}$. An analytic self-map φ of $\mathbb{D}$ defines a continuous composition operator $C_\varphi : H(\mathbb{D}) \to H(\mathbb{D})$, $C_\varphi(f) := f \circ \varphi$ for each $f \in H(\mathbb{D})$. The monographs of Cowen and MacCluer [6] and Shapiro [13] are standard references for composition operators on spaces of analytic functions; see also [2], [3], [4], [8], [14] and the references therein.

Let $T : X \to X$ be a continuous linear operator on a Fréchet space X, i.e. a complete metrizable locally convex space. We write $T \in \mathcal{L}(X)$. The *resolvent set* $\rho(T)$ of T consists of all $\lambda \in \mathbb{C}$ such that $R(\lambda, T) := (\lambda I - T)^{-1}$ is a continuous linear operator, that is $T - \lambda I : X \to X$ is bijective and has a continuous inverse. Here I stands for the identity operator on X. The set $\sigma(T) := \mathbb{C} \setminus \rho(T)$ is called the *spectrum* of T. The *point spectrum* $\sigma_{pt}(T)$ of T consists of all $\lambda \in \mathbb{C}$ such that $(\lambda I - T)$ is not injective. If we need to stress the space X, then we write $\sigma(T; X)$, $\sigma_{pt}(T; X)$ and $\rho(T; X)$. Unlike for Banach spaces X, it may happen that $\rho(T) = \emptyset$ or that $\rho(T)$ is not open. This is the reason why some authors (see e.g. [15] and [1]) prefer to consider $\rho^*(T)$ instead of $\rho(T)$ consisting of all $\lambda \in \mathbb{C}$ for which there exists $\delta > 0$ such that $B(\lambda, \delta) := \{z \in \mathbb{C} \colon |z - \lambda| < \delta\} \subset \rho(T)$ and $\{R(\mu, T) \colon\ \mu \in B(\lambda, \delta)\}$ is

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equicontinuous in $\mathcal{L}(X)$. The *Waelbroeck spectrum* is defined by $\sigma^*(T) := \mathbb{C} \setminus \rho^*(T)$. It is a closed set containing $\sigma(T)$. The reader is referred to the book of Meise and Vogt [11] for functional analysis and Fréchet spaces.

Arendt, Celariès and Chalendar [2, Theorem 4.1] prove that if $\varphi : \mathbb{D} \to \mathbb{D}$ is an analytic map with an interior fixed point $\varphi(a) = a \in \mathbb{D}$ and $0 < |\varphi'(a)| < 1$, then the spectrum of the composition operator C_φ is

$$\sigma(C_\varphi, H(\mathbb{D})) = \sigma^*(C_\varphi, H(\mathbb{D})) = \{0\} \cup \{\varphi'(a)^n \ : \ n = 0, 1, 2, ...\}.$$

Moreover, in case $\varphi'(a) = 0$, then [2, Theorem 4.10] implies that $\sigma(C_\varphi, H(\mathbb{D})) = \sigma^*(C_\varphi, H(\mathbb{D})) = \{0, 1\}$.

The eigenvalues and spectrum of composition operators on Banach spaces of analytic functions has been investigated by many authors. We refer the reader to [2], [3], [8] and [14] and the references therein.

If φ is an automorphism with an interior fixed point $\varphi(a) = a \in \mathbb{D}$, then $0 \notin \sigma(C_\varphi, H(\mathbb{D}))$, as follows for example from [2, Corollary 2.4]. Moreover, the Moebius transform $\psi_a(z) := (a - z)/(1 - \overline{a}z)$ satisfies $\psi_a(a) = 0$ and $\psi_a^{-1} = \psi_a$, and

$$\sigma(C_{\psi_a} C_\varphi C_{\psi_a}, H(\mathbb{D})) = \sigma(C_\varphi, H(\mathbb{D})).$$

Accordingly to investigate the spectrum in this case it is enough to consider the case of rotations $\varphi_r(z) := rz, z \in \mathbb{D}$, for each $r \in \mathbb{C}$ with $|r| = 1$.

In the rest of the article we denote by $H_0(\mathbb{D})$ the space of all the analytic functions on $\mathbb{D}$ such that $f(0) = 0$, which is a closed hyperplane of $H(\mathbb{D})$, hence a Fréchet space when endowed with the induced topology. Clearly $C_{\varphi_r}(H_0(\mathbb{D})) \subset H_0(\mathbb{D})$ for each $|r| = 1$. We give several results about the spectrum of the composition operator C_{φ_r} when acting on $H_0(\mathbb{D})$ which complement the aforementioned theorems in [2].

Our first result collects several general facts. Some of them might be known. They are related to the study of the translation operator on the circle, or modulo one, in ergodic theory; see e.g. [7]. From now on we denote by $\mathbb{T}$ the unit circle.

Proposition 1 *Let φ_r be the self map of $\mathbb{D}$ given by $\varphi_r(z) := rz, z \in \mathbb{D}$ for some $r \in \mathbb{T}$. Then*

(1) $\sigma_{pt}(C_{\varphi_r}, H(\mathbb{D})) = \{r^n \mid n = 0, 1, 2, ...\}$.

(2) $\sigma^*(C_{\varphi_r}, H(\mathbb{D}))$ *is contained in the unit circle* $\mathbb{T}$. *The same holds for* $\sigma^*(C_{\varphi_r}, H_0(\mathbb{D}))$.

(3) If r is a root of unity and m is the minimum positive integer such that $r^m = 1$, then

$$\sigma(C_{\varphi_r}, H(\mathbb{D})) = \sigma^*(C_{\varphi_r}, H(\mathbb{D})) = \sigma_{pt}(C_{\varphi_r}, H(\mathbb{D})) = \{1, r, r^2, ...r^{m-1}\}.$$

Moreover $\mathrm{Ker}(C_{\varphi_r} - r^j I)$ *is infinite dimensional for each $j = 0, ..., m - 1$. The same result holds for $H_0(\mathbb{D})$ instead of $H(\mathbb{D})$.*

(4) If r is not a root of unity, then $\sigma_{pt}(C_{\varphi_r}, H_0(\mathbb{D})) = \{r^n \mid n = 1, 2, ...\}$ *and* $\overline{\sigma(C_{\varphi_r}, H_0(\mathbb{D}))} = \sigma^*(C_{\varphi_r}, H_0(\mathbb{D})) = \mathbb{T}$. *The last equality of sets also holds for $H(\mathbb{D})$.*

Proof. (1) Clearly $C_{\varphi_r}(z^n) = r^n z^n$ for each $n = 0, 1, 2, \ldots$. On the other hand, if $f(z) = \sum_{n=0}^{\infty} a_n z^n \in H(\mathbb{D})$, $f \neq 0$, satisfies $f(rz) = \mu f(z)$ for each $z \in \mathbb{D}$, we have $a_n(r^n - \mu) = 0$ for each $n = 0, 1, 2, \ldots$. Since $f \neq 0$, this implies that $\mu = r^n$ for some n.

(2) We first show that $C_{\varphi_r} - \eta I : H(\mathbb{D}) \to H(\mathbb{D})$ is bijective for each $\eta \notin \mathbb{T}$, hence an isomorphism by the closed graph theorem. By part (1), $C_{\varphi_r} - \eta I$ is injective. We show that it is also surjective. Given $g(z) = \sum_{n=0}^{\infty} a_n z^n \in H(\mathbb{D})$, define $f(z) := \sum_{n=0}^{\infty} \frac{a_n}{r^n - \eta} z^n$. There is $\delta > 0$ such that $|\eta - r^n| > \delta$ for each $n = 0, 1, 2, \ldots$ Since the series $\sum_{n=0}^{\infty} |a_n||z|^n$ converges for each $|z| < 1$, we get that $\sum_{n=0}^{\infty} \frac{|a_n|}{|r^n - \eta|}|z|^n$ also converges for each $|z| < 1$, and $f(z)$ is an analytic function on $\mathbb{D}$ such that $(C_{\varphi_r} - \eta I)f = g$.

In particular we have shown that

$$(C_{\varphi_r} - \eta I)^{-1}\left(\sum_{n=0}^{\infty} a_n z^n\right) = \sum_{n=0}^{\infty} \frac{a_n}{r^n - \eta} z^n.$$

for each $\eta \notin \mathbb{T}$ and each $\sum_{n=0}^{\infty} a_n z^n \in H(\mathbb{D})$.

Now fix $\mu \notin \mathbb{T}$ and select $\delta > 0$ such that $|\eta - r| > \delta$ for each $r \in \mathbb{T}$ and each $\eta \in \mathbb{C}$ with $|\eta - \mu| < \delta$. To prove that $\mu \notin \sigma^*(C_{\varphi_r}, H(\mathbb{D}))$, by the uniform boundedness principle, it is enough to show that the set

$$B(g) := \{(C_{\varphi_r} - \eta I)^{-1} g \mid |\eta - \mu| < \delta\}$$

is bounded in $H(\mathbb{D})$ for each $g = \sum_{n=0}^{\infty} a_n z^n \in H(\mathbb{D})$. To see this, fix $\alpha \in]0, 1[$. We have

$$\sup_{|z| \leq \alpha} |(C_{\varphi_r} - \eta I)^{-1} g(z)| = \sup_{|z| \leq \alpha} \left| \sum_{n=0}^{\infty} \frac{a_n}{r^n - \eta} z^n \right| \leq \delta^{-1} \sum_{n=0}^{\infty} |a_n| \alpha^n.$$

The proof that $\sigma^*(C_{\varphi_r}, H_0(\mathbb{D})) \subset \mathbb{T}$ is the same.

(3) Clearly

$$\{1, r, r^2, \ldots r^{m-1}\} = \sigma_{pt}(C_{\varphi_r}, H(\mathbb{D})) \subset \sigma(C_{\varphi_r}, H(\mathbb{D})) \subset \sigma^*(C_{\varphi_r}, H(\mathbb{D})).$$

The proof each $\mu \notin \{1, r, r^2, \ldots r^{m-1}\}$ does not belong to $\sigma^*(C_{\varphi_r}, H(\mathbb{D}))$ is similar to the one in part (2), just taking $\delta > 0$ such that $|\eta - r^n| > \delta$ for each $n = 0, \ldots, m-1$ and each $\eta \in \mathbb{C}$ with $|\eta - \mu| < \delta$.

The space $\mathrm{Ker}(C_{\varphi_r} - r^j I), j = 0, \ldots, m-1$, of eigenvectors is infinite dimensional because $C_{\varphi_r}(z^{km+j}) = r^j z^{km+j}$ for each $k \in \mathbb{N}$.

In the case of $H_0(\mathbb{D})$, observe that, although the constants do not belong to this space, we have $C_{\varphi_r}(z^{km}) = z^{km}$ for each $k \in \mathbb{N}$. The rest of the proof is similar.

(4) If r is not a root of unity, then $r^n \neq 1$ for each $n \in \mathbb{N}$. Since the constants do not belong to $H_0(\mathbb{D})$, the argument in part (1) yields $\sigma_{pt}(C_{\varphi_r}, H_0(\mathbb{D})) = \{r^n \mid n \in \mathbb{N}\}$.

Therefore, by part (2),

$$\{r^n \mid n \in \mathbb{N}\} \subset \sigma(C_{\varphi_r}, H_0(\mathbb{D})) \subset \sigma^*(C_{\varphi_r}, H_0(\mathbb{D})) \subset \mathbb{T}.$$

By Kronecker's Theorem [12, Theorem 2.2.4], which can also be deduced from [9, Theorem 438], $\{r^n \mid n \in \mathbb{N}\}$ is dense in $\mathbb{T}$. This implies

$$\overline{\sigma(C_{\varphi_r}, H_0(\mathbb{D}))} = \sigma^*(C_{\varphi_r}, H_0(\mathbb{D})) = \mathbb{T}.$$

The proof that $\sigma^*(C_{\varphi_r}, H(\mathbb{D})) = \mathbb{T}$ is obtained with the same argument. $\square$

There are examples of continuous linear operators T on a Fréchet space E such that $\overline{\sigma(T,E)}$ is properly contained in $\sigma^*(T,E)$. See Remark 3.5 (vi) in [1]. Compare with the statement in Proposition 1 (4).

As a consequence of Proposition 1 (4), if X is a Banach space of analytic functions containing the polynomials which is continuously included in $H(\mathbb{D})$ and such that rotations define continuous composition operators on X, then $\sigma(C_{\varphi_r}, X) = \mathbb{T}$. We see below that the behaviour in the Fréchet space $H_0(\mathbb{D})$ is different; in particular $\sigma(C_{\varphi_r}, H_0(\mathbb{D}))$ need not be closed.

Theorem 2 *Let $r \in \mathbb{T}$ be such that $r^n \neq 1$ for each $n \in \mathbb{N}$ and let $\varphi_r(z) := rz, z \in \mathbb{D}$. Let $\lambda \in \mathbb{T}$ satisfy $\lambda \neq r^n$ for each $n \in \mathbb{N}$. The following conditions are equivalent:*

(i) $\lambda \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$.

(ii) For each $\alpha \in \,]0,1[$ there exists $\delta(\alpha) > 0$ such that $|r^n - \lambda| \geq \delta(\alpha)\alpha^n$ for each $n \in \mathbb{N}$.

Proof. By the closed graph theorem, condition (i) holds if and only if $C_{\varphi_r} - \lambda I : H_0(\mathbb{D}) \to H_0(\mathbb{D})$ is bijective. Since $\lambda \neq r^n$ for each $n \in \mathbb{N}$, $C_{\varphi_r} - \lambda I$ is injective by Proposition 1.

(ii) implies (i). It is enough to show that condition (ii) implies that $C_{\varphi_r} - \lambda I : H_0(\mathbb{D}) \to H_0(\mathbb{D})$ is surjective. Fix $g(z) = \sum_{n=1}^{\infty} a_n z^n \in H_0(\mathbb{D})$ and define $f(z) := \sum_{n=1}^{\infty} \frac{a_n}{r^n - \lambda} z^n$. Fix $\alpha \in \,]0,1[$ and select $\beta \in \,]\alpha, 1[$. By (ii), there is $\delta(\beta) > 0$ such that $|r^n - \lambda| \geq \delta(\beta)\beta^n$ for each $n \in \mathbb{N}$. For each $|z| \leq \alpha$ we have

$$\sum_{n=1}^{\infty} \frac{|a_n|}{|r^n - \lambda|} |z|^n \leq \frac{1}{\delta(\beta)} \sum_{n=1}^{\infty} |a_n| \frac{\alpha^n}{\beta^n} < \infty.$$

Therefore $f \in H_0(\mathbb{D})$ and, clearly, $(C_{\varphi_r} - \lambda I)f = g$.

(i) implies (ii). If $\lambda \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$, then $C_{\varphi_r} - \lambda I : H_0(\mathbb{D}) \to H_0(\mathbb{D})$ is surjective. Given $\sum_{n=1}^{\infty} z^n \in H_0(\mathbb{D})$, its pre-image by $C_{\varphi_r} - \lambda I$ is $\sum_{n=1}^{\infty} \frac{1}{r^n - \lambda} z^n$ and it belongs to $H_0(\mathbb{D})$. In particular, for each $\alpha \in]0,1[$,

$$\sum_{n=1}^{\infty} \frac{1}{|r^n - \lambda|} \alpha^n < \infty.$$

Therefore $\sup_{n \in \mathbb{N}} \frac{1}{|r^n - \lambda|} \alpha^n < \infty$, which clearly implies (ii). $\square$

A real number $x \in \mathbb{R}$ is called *Diophantine* if there are $\delta \geq 1$ and $c(x) > 0$ such that

$$\left| x - \frac{p}{q} \right| \geq \frac{c(x)}{q^{1+\delta}}$$

for all rational numbers p/q. See [12, Definition 3.1.9]. The complement of the set of Diophantine numbers has Lebesgue measure 0; see [5, page 43]. By a Theorem of Liouville (see [9, Theorem 191] or [12, Proposition 3.1.3]) every irrational algebraic number of degree larger than or equal 2 is Diophantine. See also [10] for further information.

The proof of Theorem 4 below requires the construction of irrational numbers x with very fast Diophantine approximation. The next lemma follows from the more general result Theorem 22 in [10]. We include a proof which does not use continued fractions for the sake of completeness.

Lemma 3 *For each $m \in \mathbb{N}, m \geq 2$, there is a positive real number $x(m)$ and sequences of positive numbers $(p_j)_{j=1}^{\infty}$ and $(q_j)_{j=1}^{\infty}$ with $\lim_{j\to\infty} q_j = +\infty$ such that*

$$\left|x(m) - \frac{p_j}{q_j}\right| \leq (\frac{1}{m})^{q_j}$$

Proof. Given $m \in \mathbb{N}, m \geq 2$, define inductively $q_1 := m$ and $q_{j+1} := (q_j)^{q_j}$ for $j \in \mathbb{N}$. Clearly $q_j \geq m^j$ and q_j is a divisor of q_{j+1} for each $j \in \mathbb{N}$. We set $x(m) := \sum_{j=1}^{\infty} 1/q_j$. The series converges, and each partial sum $\sum_{j=1}^{k} 1/q_j$ is a rational number which can be written of the form p_k/q_k for some $p_k \in \mathbb{N}$. Moreover, we have

$$\left|x(m) - \frac{p_k}{q_k}\right| = \sum_{j=k+1}^{\infty} \frac{1}{q_j} \leq \frac{1}{q_{k+1}} \sum_{j=0}^{\infty} \left(\frac{1}{m}\right)^j \leq \frac{2}{q_{k+1}}.$$

Since $2m^{q_k} \leq q_{k+1}$ for each $k \in \mathbb{N}$, we have $2/q_{k+1} \leq (1/m)^{q_k}$ for each $k \in \mathbb{N}$. Therefore the rational numbers $p_k/q_k, k \in \mathbb{N}$, satisfy

$$\left|x(m) - \frac{p_k}{q_k}\right| \leq \left(\frac{1}{m}\right)^{q_k}$$

for all $k \in \mathbb{N}$. □

Theorem 4 *(1) If $x \in \mathbb{R}$ is a Diophantine number, then the rotation $\varphi_r(z) = rz, z \in \mathbb{D}$, for $r := e^{i2\pi x}$ satisfies that $1 \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$. In particular, $1 \in \sigma^*(C_{\varphi_r}, H_0(\mathbb{D}))\backslash\sigma(C_{\varphi_r}, H_0(\mathbb{D}))$, and $\sigma(C_{\varphi_r}, H_0(\mathbb{D}))$ is not closed.*

(2) Let $x \in \mathbb{R}$ be an irrational number such that there are $\alpha \in]0,1[$ and a sequence of rational numbers $(p_j/q_j)_{j=1}^{\infty}$ with $\lim_{j\to\infty} q_j = +\infty$ such that

$$\left|x - \frac{p_j}{q_j}\right| \leq \alpha^{q_j},$$

for each $j \in \mathbb{N}$. Then the rotation $\varphi_r(z) := rz, z \in \mathbb{D}$, for $r := e^{i2\pi x}$ satisfies $1 \in \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$.

Proof. (1) If x is a Diophantine number, then there are $d > 0$ and $\delta \geq 1$ such that $r := e^{i2\pi x}$ satisfies $|r^n - 1| \geq d/n^{\delta}$ for all $n \in \mathbb{N}$, [5, page 43]. Hence there is $D > 0$ such that $1/|r^n - 1| \leq Dn^{\delta}$ for each $n \in \mathbb{N}$. Given $\alpha \in]0,1[$, for each $n \in \mathbb{N}$, we have

$$\frac{1}{|r^n - 1|}\alpha^n \leq Dn^{\delta}\alpha^n.$$

This implies $\sup_{n\in\mathbb{N}} \frac{1}{|r^n-1|}\alpha^n < \infty$, which implies $1 \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$ by Theorem 2.

The spectrum $\sigma(C_{\varphi_r}, H_0(\mathbb{D}))$ is not closed in this case, because it contains $\{r^n \mid n \in \mathbb{N}\}$, which is dense in $\mathbb{T}$ by Kronecker's Theorem, but $1 \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$.

(2) The complex number $r := e^{i2\pi x}$ satisfies, for each $j \in \mathbb{N}$,

$$|r^{q_j} - 1| = |e^{i2\pi q_j x} - e^{i2\pi p_j}| \leq 2\pi|q_j x - p_j| \leq 2\pi q_j \alpha^{q_j}.$$

Proceeding by contradiction, assume that $1 \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$. Select any $\beta \in]\alpha,1[$. By Theorem 2, there is $\delta(\beta) > 0$ such that $|r^n - 1| \geq \delta(\beta)\beta^n$ for each $n \in \mathbb{N}$. Specializing for $n = q_j, j \in \mathbb{N}$, we get

$$\delta(\beta)\beta^{q_j} \leq |r^{q_j} - 1| \leq 2\pi q_j \alpha^{q_j} \quad \text{for each} \quad j \in \mathbb{N}.$$

This is a contradiction, since $0 < \alpha < \beta$ and q_j tends to infinity as j goes to infinity. □

Corollary 5 *There exist $r, s \in \mathbb{T}$ such that the corresponding rotations φ_r and φ_s satisfy $1 \notin \sigma(C_{\varphi_r}, H_0(\mathbb{D}))$ and $1 \in \sigma(C_{\varphi_s}, H_0(\mathbb{D}))$.*

Proof. This is a direct consequence of Theorem 4 since Lemma 3 ensures the existence of irrational numbers satisfying the hypothesis of part (2) in the Theorem. □

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