

Isometric weighted composition operators on weighted Banach spaces of type H^∞

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Abstract

We characterize those weighted composition operators on weighted Banach spaces of holomorphic functions of type H^∞ which are an isometry.

Introduction

Let v be a strictly positive continuous function (*weight*) on the open unit disk D in the complex plane which is radial (that is, $v(z) = v(|z|)$ for every $z \in D$) and decreasing with respect to $|z|$. We are interested in operators defined on the weighted Banach spaces of holomorphic functions

$$H_v^\infty := \{f \in H(D); \|f\|_v = \sup_{z \in D} v(z)|f(z)| < \infty\}$$

and on the smaller spaces

$$H_v^0 := \{f \in H_v^\infty, \lim_{|z| \rightarrow 1} v(z)|f(z)| = 0\},$$

both endowed with the norm $\|\cdot\|_v$. Here $H(D)$ denotes the space of all analytic functions. Spaces of this type appear in the study of growth conditions of analytic functions and have been investigated in various articles, see e.g. [10], [1], [14], [13], [2], [3], [4]. We consider non-constant functions $\varphi \in H(D)$ satisfying $\varphi(D) \subset D$ as well as functions $\psi \in H(D)$ which are not identically equal zero. These functions induce the weighted composition operator $C_{\varphi, \psi}$ which is a linear map on $H(D)$ and given by $C_{\varphi, \psi} = \psi(f \circ \varphi)$. If $\psi = 1$, then as usual we denote $C_{\varphi, 1}$ by C_φ .

In [16] M.J. Martin and D. Vukotic analyze when composition operators on the Bloch space are a (not necessarily surjective) isometry, and they show that every thin Blaschke product induces an isometric composition operator on the Bloch space. The same authors give a unified proof of the characterization of all possible composition operators that are isometries on some classical function spaces, see [15]. Laitila [11] characterizes analytic self maps φ which induce isometric composition operators on the space BMOA equipped with a Möbius invariant H^2 norm. A good general reference on isometries of spaces of analytic functions is the book of R. Fleming and J. Jamison [8].

In this note our aim is to characterize isometric weighted composition operators on weighted Banach spaces of holomorphic functions as defined above, thus extending the result of M.J. Martin and D. Vukotic in [16]. Our main results are Theorem 2 and Theorem 9.

Notation and preliminaries

For notation and more information on composition operators we refer the reader to the excellent monographs [6] and [19]. In order to formulate results on weighted spaces of analytic functions we need the so called *associated weights* (see [2]). For a weight v the associated weight \tilde{v} is given by

$$\tilde{v}(z) := \frac{1}{\sup\{|f(z)|; f \in H_v^\infty, \|f\|_v \leq 1\}} = \frac{1}{\|\delta_z\|_{H_v^\infty}}, \quad z \in D,$$

where δ_z denotes the point evaluation of z . By [2] we know that associated weights are continuous, $\tilde{v} \geq v > 0$ and that for each $z \in D$ we can find $f_z \in H_v^\infty$, $\|f_z\|_v \leq 1$, such that $|f_z(z)| = \frac{1}{\tilde{v}(z)}$. It

is well known that H_v^∞ is isometrically isomorphic to H_v^∞ . We are especially interested in radial weights which satisfy the following condition which is due to W. Lusky (see [13])

$$(L1) \quad \inf_k \frac{v(1-2^{-k-1})}{v(1-2^{-k})} > 0.$$

The standard weights $v_p(z) = (1-|z|^2)^p$, $p > 0$, are weights which have (L1).

We also need some geometric data of the unit disk. The *pseudohyperbolic metric* is given by

$$\rho(z, a) := |\sigma_a(z)|, \text{ where } \sigma_a(z) := \frac{a-z}{1-\bar{a}z}$$

is the automorphism of \mathbb{D} which changes 0 and a . Furthermore, we will use the following equality which is easy to check

$$1 - |\sigma_a(z)|^2 = \frac{(1-|a|^2)(1-|z|^2)}{|1-\bar{a}z|^2}.$$

Isometric weighted composition operators

The surjective isometries on the spaces $H_{v_p}^\infty$, $0 < p < \infty$, can be characterized following the argument given by J. Cima and W. Wogen in [7],[8] for $H_{v_1}^\infty$. More precisely, *if $S : H_{v_p}^\infty \rightarrow H_{v_p}^\infty$ is a surjective isometry, then there is a conformal automorphism φ of D and $\lambda \in \partial D$ such that $Sf = C_{\varphi, \lambda(\varphi)'} f$ for every $f \in H_{v_p}^\infty$* . Even for the standard weights a characterization of all isometries on these spaces does not seem to be known. But in the case of the smaller space $H_{v_p}^0$ all isometries can be described in this way (see [7],[8]). Therefore we will focus our attention on isometries among the weighted composition operators acting on H_v^∞ for more general weights v .

The following result which is crucial for the proof of our main result, can be found in [12] (see [9, Lemma 5.1] for the case of standard weights v_p).

Lemma 1 *Let v be a radial weight with (L1) such that v is continuously differentiable with respect to $|z|$. Then there is $M > 0$ such that for $f \in H_v^\infty$ we have*

$$|v(p)f(p) - v(q)f(q)| \leq M\|f\|_v \rho(p, q)$$

for all $p, q \in D$.

Theorem 2 *Let φ be an analytic self map of D and $\psi \in H(D)$.*

- (a) *Assume that v is a radial weight with (L1) such that v is continuously differentiable with respect to $|z|$. If $\sup_{z \in D} \frac{|\psi(z)|v(z)}{\tilde{v}(\varphi(z))} \leq 1$ and the following condition holds*
 $(M_{\varphi, \psi, v})$ *for every $a \in D$ there exists a sequence $(z_n)_n \subset D$ such that*

$$\rho(\varphi(z_n), a) \rightarrow 0 \quad \text{and} \quad \frac{|\psi(z_n)|v(z_n)}{\tilde{v}(\varphi(z_n))} \rightarrow 1,$$

then $C_{\varphi, \psi} : H_v^\infty \rightarrow H_v^\infty$ is an isometry.

- (b) *Let v be a radial weight such that $w(z) := \frac{v(z)}{(1-|z|^2)^p}$ is a weight on D for some $0 < p < \infty$ and $w = \tilde{w}$. If $C_{\varphi, \psi} : H_v^\infty \rightarrow H_v^\infty$ is an isometry, then condition $(M_{\varphi, \psi, v})$ is satisfied and $\sup_{z \in D} \frac{|\psi(z)|v(z)}{\tilde{v}(\varphi(z))} \leq 1$.*

Proof. (a) By assumption, for each $f \in H_v^\infty$ we have that

$$\|C_{\varphi, \psi} f\|_v = \sup_{z \in D} \frac{|\psi(z)|v(z)}{\tilde{v}(\varphi(z))} |\tilde{v}(\varphi(z))f(\varphi(z))| \leq \|f\|_v.$$

In order to verify the reverse inequality, let $f \in H_v^\infty$. Then $\|f\|_v = \lim_{m \rightarrow \infty} v(a_m)|f(a_m)|$ for some sequence $(a_m)_m$. Let m be fixed. Hence, by condition $(M_{\varphi, \psi, v})$, there is a sequence $(z_n^m)_n \subset D$ such that $\rho(\varphi(z_n^m), a_m) \rightarrow 0$ and $\frac{|\psi(z_n^m)|v(z_n^m)}{\tilde{v}(\varphi(z_n^m))} \rightarrow 1$, when $n \rightarrow \infty$. By Lemma 1, for all m and n ,

$$|v(a_m)f(a_m) - v(\varphi(z_n^m))f(\varphi(z_n^m))| \leq M\|f\|_v\rho(a_m, \varphi(z_n^m)).$$

Hence

$$\begin{aligned} \|C_{\varphi, \psi}f\|_v &= \sup_{z \in D} \frac{|\psi(z)|v(z)}{\tilde{v}(\varphi(z))} \tilde{v}(\varphi(z))|f(\varphi(z))| \\ &\geq \limsup_{n \rightarrow \infty} \frac{|\psi(z_n^m)|v(z_n^m)}{\tilde{v}(\varphi(z_n^m))} (|f(a_m)|v(a_m) - M\|f\|_v\rho(\varphi(z_n^m), a_m)) = v(a_m)|f(a_m)|. \end{aligned}$$

Since this is true for all m , $\|C_{\varphi, \psi}f\|_v \geq \|f\|_v$.

(b) Choose $p > 0$ such that $w(z) := \frac{v(z)}{(1-|z|^2)^p}$ is a weight on D with $\tilde{w} = w$. First we want to show that this implies $v = \tilde{v}$. Since $v(z) \leq \tilde{v}(z)$ for each $z \in D$, it is enough to show the reverse inequality. Fix $a \in D$, and select $k_a \in H(D)$ with $|k_a| \leq 1/w$ and $|k_a(a)| = 1/w(a)$. If $a \neq 0$, select b with $|b| = 1$ and $a = |a|b$ and define $h_a(z) := k_a(z)/(1-(\bar{b}z)^2)^p$, $z \in D$. Clearly $|h_a| \leq 1/\tilde{v}$ on D , and $|h_a(a)| = 1/v(a)$, thus $1/v(a) \leq 1/\tilde{v}(a)$, as desired. In case $a = 0$, it is enough to take $h_0(z) := k_0(z)$, $z \in D$, to reach the conclusion.

By assumption, $\|C_{\varphi, \psi}f\|_v = \|f\|_v$ for all $f \in H_v^\infty$. Hence

$$\|C_{\varphi, \psi}\| = \sup_{z \in D} \frac{|\psi(z)|v(z)}{\tilde{v}(\varphi(z))} \leq 1.$$

Let $a \in D$. Then there exists $g_a \in H_w^\infty(D)$, $\|g_a\|_w = 1$ with $g_a(a)\tilde{w}(a) = 1$. Put

$$f_a(z) = g_a(z) \left(\frac{1-|a|^2}{(1-z\bar{a})^2} \right)^p.$$

It follows that $\|f_a\|_v = 1$ since $|f_a(a)|v(a) = 1$. Thus we can pick a sequence $(z_n)_n \subset D$ so that $|\psi(z_n)||f_a(\varphi(z_n))|v(z_n) \rightarrow 1$ when $n \rightarrow \infty$.

Hence

$$1 \geq \frac{|\psi(z_n)|v(z_n)}{\tilde{v}(\varphi(z_n))} \geq \frac{|\psi(z_n)|v(z_n)}{\tilde{v}(\varphi(z_n))} |f_a(\varphi(z_n))| \tilde{v}(\varphi(z_n)) = |\psi(z_n)|v(z_n)|f_a(\varphi(z_n))|,$$

so

$$\lim_{n \rightarrow \infty} \frac{|\psi(z_n)|v(z_n)}{\tilde{v}(\varphi(z_n))} = 1.$$

Further,

$$\begin{aligned} 1 &\geq (1 - |\sigma_a(\varphi(z_n))|)^2)^p = \frac{(1-|a|^2)^p(1-|\varphi(z_n)|^2)^p}{|1-\varphi(z_n)\bar{a}|^{2p}} \\ &= \frac{|f_a(\varphi(z_n))|v(\varphi(z_n))(1-|\varphi(z_n)|^2)^p}{|g_a(\varphi(z_n))|v(\varphi(z_n))} \geq |f_a(\varphi(z_n))|v(\varphi(z_n)). \end{aligned}$$

Since $|f_a(\varphi(z_n))|\tilde{v}(\varphi(z_n)) \rightarrow 1$, when $n \rightarrow \infty$, we conclude, as $v = \tilde{v}$, that $\lim_{n \rightarrow \infty} (1 - |\sigma_a(\varphi(z_n))|)^2)^p = 1$ and $\rho(\varphi(z_n), a) \rightarrow 0$ when $n \rightarrow \infty$. \square

Example 3 Besides the standard weights v_p the following weights also satisfy the assumptions in Theorem 2 (b);

$$v(z) = (1-|z|^2)^p(1-\log(1-|z|^2))^{-\beta}, \quad 0 < p < \infty \text{ and } \beta > 0.$$

Example 4 (i) Fix $b \in D$ and $0 < p < \infty$. Put $\varphi(z) := \sigma_b(z)$ and $\psi(z) := (\sigma'_b)^p(z)$. Then an easy calculation shows that $C_{\varphi, \psi} : H_{v_p}^\infty \rightarrow H_{v_p}^\infty$ satisfies condition (M_{φ, ψ, v_p}) as well as $\sup_{z \in D} \frac{|\psi(z)v_p(z)}{v_p(\varphi(z))} \leq 1$, so by Theorem 2 it is an isometry, but 0 is not a fixed point of the self map.

(ii) Analogously to the proof of [16] Theorem 1.2 we can show that for each thin Blaschke product B such that $B(0) = 0$ the weighted composition operator $C_{B, (B')^p} : H_{v_p}^\infty \rightarrow H_{v_p}^\infty$ is an isometry for every $0 < p < \infty$.

Let B_p , $0 < p < \infty$, denote the Bloch type spaces of functions $f \in H(D)$ with $f(0) = 0$ satisfying $\|f\|_p := \sup_{z \in D} (1 - |z|^2)^p |f'(z)| < \infty$. Then $\|\cdot\|_p$ becomes a norm and B_p a Banach space. The map $S_p : B_p \rightarrow H_{v_p}^\infty$ given by $S_p(f) = f'$ is a surjective isometry.

Since $C_\varphi = S_p^{-1} \circ C_{\varphi, \varphi'} \circ S_p$, we have $C_\varphi : B_p \rightarrow B_p$ is an isometry if and only if $C_{\varphi, \varphi'} : H_{v_p}^\infty \rightarrow H_{v_p}^\infty$ is as well. By Theorem 2 we get the following result for Bloch type spaces.

Corollary 5 *Let φ be an analytic self map of D and $0 < p < \infty$. The operator $C_\varphi : B_p \rightarrow B_p$ is an isometry if and only if $\sup_{z \in D} \frac{|\varphi'(z)v_p(z)}{v_p(\varphi(z))} \leq 1$ and the following condition holds $(M_{\varphi, \varphi', v_p})$ for every $a \in D$ there exists a sequence $(z_n)_n \subset D$ such that*

$$\rho(\varphi(z_n), a) \rightarrow 0 \quad \text{and} \quad \frac{|\varphi'(z_n)v_p(z_n)}{v_p(\varphi(z_n))} \rightarrow 1.$$

Let \mathcal{B} denote the classical Bloch space of all analytic functions f with the norm $\|f\|_{\mathcal{B}} = |f(0)| + \|f\|_1 < \infty$. If $\varphi(0) = 0$, then clearly $C_\varphi : \mathcal{B} \rightarrow \mathcal{B}$ is an isometry if and only if $C_\varphi : B_1 \rightarrow B_1$ is an isometry. From $\|C_\varphi f\|_{\mathcal{B}} = \|f\|_{\mathcal{B}}$ for all $f \in \mathcal{B}$, it follows that $|f(\varphi(0))| \geq |f(0)|$ for all $f \in \mathcal{B}$. By choosing $f(z) := \frac{\varphi(0) - z}{1 - \overline{\varphi(0)}z} \in \mathcal{B}$, one gets $\varphi(0) = 0$ (see [15]). Therefore we obtain Theorem 1.1 in [16].

Corollary 6 *Let φ be an analytic self map of D and $0 < p < \infty$. Then $C_\varphi : \mathcal{B} \rightarrow \mathcal{B}$ is an isometry if and only if $\varphi(0) = 0$ and the following condition holds $(M_{\varphi, \varphi', v_1})$ for every $a \in D$ there exists a sequence $(z_n)_n \subset D$ such that*

$$\rho(\varphi(z_n), a) \rightarrow 0 \quad \text{and} \quad \frac{|\varphi'(z_n)|(1 - |z_n|^2)}{1 - |\varphi(z_n)|^2} \rightarrow 1.$$

Now we restrict our attention to (non weighted) composition operators, and obtain a result that should be compared with M.J. Martin, D. Vukotic [15] Theorem 1.3(b).

If v is radial there are only two possibilities: Either $\tilde{v}(r)$ tends to 0 as r tends to 1 or it tends to a positive number. In the second case $H_v^\infty = H^\infty$. This is why we consider first the case H^∞ , which is essentially known. Let S be any isometry of H^∞ onto H^∞ . Then S has the form $Sf = \lambda C_\varphi f$ for every $f \in H^\infty$, where φ is a conformal automorphism of D and $\lambda \in \partial D$ (see [8, Theorem 4.2.2]). In [17] R. Roan shows that $C_\varphi : H^\infty \rightarrow H^\infty$ has closed range if and only if $\partial D \subset \overline{\varphi(D)}$. His argument also characterizes isometry of C_φ on H^∞ . Indeed, by using Lemma 3 in D. Sarason [18], one concludes that $\partial D \subset \overline{\varphi(D)}$ implies

$$\{z \in D; |f(z)| \leq \sup_{w \in \varphi(D)} |f(w)| \forall f \in H^\infty\} = D.$$

Thus, for $f \in H^\infty$

$$\|f\|_\infty \geq \|C_\varphi f\|_\infty = \sup_{z \in D} |f(\varphi(z))| \geq \|f\|_\infty.$$

Theorem 7 *Let $\varphi : D \rightarrow D$ be an analytic self map. The following statements are equivalent:*

(i) $C_\varphi : H^\infty \rightarrow H^\infty$ is an isometry.

(ii) $C_\varphi : H^\infty \rightarrow H^\infty$ has closed range.

(iii) $\partial D \subset \overline{\varphi(D)}$.

Example 8 By [8] $\varphi : D \rightarrow D, \varphi(z) = \frac{-1+2(1+iz)}{1+iz+\sqrt{2(1-z^2)}}$ is not a surjection, but satisfies $\overline{\varphi(D)} = \overline{D}$ and therefore C_φ is an isometry on H^∞ .

We treat now isometric composition operators on H_v^∞ when the weight v satisfies that $\tilde{v}(r)$ tends to 0 as r tends to 1.

Theorem 9 Let v be a radial weight such that $\lim_{r \rightarrow 1} \tilde{v}(r) = 0$. Let φ be an analytic self map on D such that $\varphi(0) = 0$. The composition operator C_φ on H_v^∞ is an isometry if and only if φ is a rotation.

Proof. If φ is a rotation, then C_φ is an isometry, since v is radial. Conversely, since $\varphi(0) = 0$, Schwarz lemma gives $|\varphi(z)| \leq |z|$ for each $z \in D$. Set $\tilde{v}(1) := 0$. Recall that \tilde{v} is radial and continuous. It is enough to find $a \in D, a \neq 0$, with $|\varphi(a)| = |a|$. To do this, find $b \in]0, 1[$ with $\tilde{v}(b)b = \max_{0 \leq r \leq 1} r\tilde{v}(r)$, and $\tilde{v}(z)|z| < \tilde{v}(b)b$ if $b < |z| < 1$. Select $b < c < 1$ and find $d \in [c, 1[$ with $d\tilde{v}(d) = \max_{c \leq x \leq 1} x\tilde{v}(x)$. Clearly $d\tilde{v}(d) < b\tilde{v}(b)$, by the selection of b . For $|z| \geq c$, we have $\tilde{v}(z)|\varphi(z)| \leq \tilde{v}(z)|z| \leq \tilde{v}(d)d < \tilde{v}(b)b$. Since C_φ is an isometry, $\sup_{z \in D} \tilde{v}(z)|\varphi(z)| = \tilde{v}(b)b$. Find a sequence $(z_j)_j$ in D such that $|z_j| \leq c$ for all j and $\tilde{v}(z_j)|\varphi(z_j)|$ tends to $\tilde{v}(b)b$. Consequently, the sequence has an accumulation point a with $|a| \leq c$. This yields,

$$\sup_{z \in D} \tilde{v}(z)|\varphi(z)| = \tilde{v}(a)|\varphi(a)| = \tilde{v}(b)b = \sup_{z \in D} \tilde{v}(z)|z| > 0.$$

In particular $a \neq 0$, since otherwise $\varphi(a) = 0$. This implies $\tilde{v}(a)|\varphi(a)| \geq \tilde{v}(a)|a|$, hence $|\varphi(a)| = |a|$. The proof is complete. \square

Proposition 10 Let v be a radial weight such that its associated weight \tilde{v} is strictly decreasing on $[0, 1[$. Let φ be an analytic self map on D . If the composition operator C_φ is an isometry on $H_v^\infty(D)$, then $\varphi(0) = 0$.

Proof. Recall that H_v^∞ and $H_{\tilde{v}}^\infty$ coincide isometrically. Fix $a \in D$. Select $f \in H_v^\infty$ such that $|f| \leq 1/v, |f(\varphi(a))| = 1/\tilde{v}(\varphi(a))$. Clearly $\|f\|_v = 1 = \tilde{v}(\varphi(a))|f(\varphi(a))|$. This implies

$$1 = \|C_\varphi(f)\|_v \geq \tilde{v}(a)|f(\varphi(a))| = \tilde{v}(a)/\tilde{v}(\varphi(a)).$$

This yields $\tilde{v}(a) \leq \tilde{v}(\varphi(a))$, and, since \tilde{v} is strictly decreasing, $|\varphi(a)| \leq |a|$. Applying this for $a = 0$, we conclude $\varphi(0) = 0$. \square

Remark 11 Let φ be an analytic self map of D and $0 < p < \infty$. Then φ is a rotation if and only if $\varphi(0) = 0$ and the following property is valid:

$(M_{\varphi,1,v_p})$ for every $a \in D$ there exists a sequence $(z_n)_n \subset D$ such that

$$\rho(\varphi(z_n), a) \rightarrow 0 \quad \text{and} \quad \frac{v_p(z_n)}{v_p(\varphi(z_n))} \rightarrow 1.$$

Indeed, suppose that $\varphi(0) = 0$ and $(M_{\varphi,1,v_p})$ holds. Then it is enough to prove that there is $b \in D, b \neq 0$, with $|\varphi(b)| = |b|$. Let $a \neq 0, a \in D$. By assumption we can find a sequence $(z_n)_n \subset D$ such that $\rho(\varphi(z_n), a) \rightarrow 0$ and $\frac{v_p(z_n)}{v_p(\varphi(z_n))} \rightarrow 1$. Hence $|z_n| \rightarrow |a|$, when $n \rightarrow \infty$. Now there exists a subsequence (z_m) of $(z_n)_n$ such that $z_m \rightarrow b$ with $|b| \leq 1$. Therefore $|a| = |b| < 1$ and

$$\| |\varphi(b)| - |b| \| = \| |\varphi(b)| - |a| \| \leq 2\rho(\varphi(b), a) = 2 \lim_m \rho(\varphi(z_m), a) = 0,$$

from which we conclude that $|\varphi(b)| = |b|$. The other direction is obvious.

Our last result is a consequence of Theorem 9 and Proposition 10.

Corollary 12 *Let φ be an analytic self map on \mathbb{D} and $p > 0$. The composition operator C_φ on $H_{v_p}^\infty$ is an isometry if and only if φ is a rotation.*

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