Weighted (LB)-spaces of holomorphic functions

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joint work with Klaus D. Bierstedt

Let G be an open subset of $\mathbb{C}^N$.

- $H(G)$ is the space of all holomorphic functions on G.

- A **weight** v on G is a strictly positive continuous function on G.

- For such a weight, the **weighted Banach spaces of holomorphic functions on** G are defined by

$$Hv(G) := \{f \in H(G);\ ||f||_v = \sup_{z \in G} v(z)|f(z)| < +\infty\},$$

$$Hv_0(G) := \{f \in H(G);\ vf \text{ vanishes at } \infty \text{ on } G\},$$

endowed with the norm $||.||_v$.

- For a weight v on G, the **associated weight** $\tilde{v}$ is defined by

$$\tilde{v}(z) := \frac{1}{||\delta_z||} = \frac{1}{\sup\{|f(z)|;\ f \in B_v\}},\ z \in G,$$

where $\delta_z \in Hv(G)'$ denotes the point evaluation in $z \in G$.

Let G be an open subset of $\mathbb{C}^N$.

1. Lemma.

(a) Let $(f_k)_k$ be a sequence in $Hv(G)$ which converges to 0 in co. If

$$(*) \qquad \begin{array}{l} \forall \varepsilon > 0 \ \exists k(\varepsilon) \in \mathbb{N}, \ K_\varepsilon \subset G \text{ compact } \forall k \geq k(\varepsilon) \\ \forall z \in G \setminus K_\varepsilon : \ w(z)|f_k(z)| < \varepsilon, \end{array}$$

$$\Downarrow$$

$$f_k \to 0 \textit{ in } Hw_0(G).$$

(b) If $(*)$ holds for each bounded sequence $(f_k)_k$ in $Hv(G)$ which tends to 0 in co, then the unit ball B_v of $Hv(G)$ is relatively compact in $Hw_0(G)$; that is, the inclusion map

$$Hv(G) \longrightarrow Hw_0(G)$$

is compact.

PROOF. (a) Fix $\varepsilon > 0$ and set $K := K_\varepsilon$. If $z \in G \backslash K$, one has $w(z)|f_k(z)| < \varepsilon$ for each $k \geq k(\varepsilon)$. On the other hand, $f_k \to 0$ holds uniformly on K; thus we find $k_0 \in \mathbb{N}$, without loss of generality $k_0 \geq k(\varepsilon)$, such that for any $k \geq k_0$ and any $z \in K$,

$$|f_k(z)| < \frac{\varepsilon}{\sup_{z \in K} w(z)}.$$

It follows that $f_k \to 0$ in $Hw_0(G)$ since for $k \geq k_0$ and $z \in K$ one easily concludes that $w(z)|f_k(z)| < \varepsilon$, while if $z \in G \setminus K$, then again $w(z)|f_k(z)| < \varepsilon$ since $k_0 \geq k(\varepsilon)$.

(b) To show that B_v is relatively compact in $Hw_0(G)$, we fix a sequence $(f_k)_k \subset B_v$ and will find a subsequence which converges in $Hw_0(G)$. Since B_v is compact in $(H(G), \mathrm{co})$ by Montel's theorem, there is a subsequence $(f_{k_j})_j$ which converges to $f_0 \in B_v$ in co. Now $(f_{k_j} - f_0)_j \subset 2B_v$ and $f_{k_j} - f_0 \to 0$ in co as $j \to \infty$. By (a) we then get $f_{k_j} - f_0 \to 0$ in $Hw_0(G)$; i.e., $f_{k_j} \to f_0$ in $Hw_0(G)$. $\square$

2. Definition.

A sequence $(z_j)_j \subset G$ is **interpolating** for $Hv(G)$ if for every sequence $(\alpha_j)_j \subset \mathbb{C}$ with

$$\sup_{j\in\mathbb{N}} v(z_j)|\alpha_j| < +\infty$$

there is $g \in Hv(G)$ such that $g(z_j) = \alpha_j$ for each $j \in \mathbb{N}$.

Let

$$\ell_\infty(v) = \ell_\infty((v(z_j))_j) := \{(\alpha_j)_j \in \mathbb{C}^{\mathbb{N}};\ \sup_j v(z_j)|\alpha_j| < +\infty\}.$$

Then $(z_j)_j$ is interpolating for $Hv(G)$ precisely if the map

$$\begin{array}{rcl} R : Hv(G) & \longrightarrow & \ell_\infty(v) \\ g & \rightsquigarrow & R(g) := (g(z_j))_j \end{array}$$

is surjective.

3. Proposition. *Assume that the set-theoretic inclusion $Hv(G) \subset Hw_0(G)$ holds. If every discrete sequence $(z_j)_j$ in G contains an interpolating subsequence for $Hv(G)$, then the inclusion map*

$$Hv(G) \longrightarrow Hw_0(G)$$

is compact.

PROOF. By Lemma 1.(b) it suffices to show that $(*)$ holds for each bounded sequence $(f_k)_k$ in $Hv(G)$ which converges to 0 with respect to co, and we fix such a sequence $(f_k)_k$. Suppose that $(*)$ does not hold. Then, for a fundamental sequence $(K_k)_k$ of compact subsets of G,

$$\exists \varepsilon_0 > 0 \ \forall k \in \mathbb{N} \ \exists z_k \in G \setminus K_k, \ z_k \neq z_j \text{ for } j < k :$$

$$w(z_k)|f_k(z_k)| \geq \varepsilon_0.$$

Since $(f_k)_k$ is bounded in $Hv(G)$,

$$\sup_k v(z_k)|f_k(z_k)| \leq \sup_k \sup_{z \in G} v(z)|f_k(z)| < +\infty$$

holds. By construction $(z_k)_k$ is discrete; hence by our hypothesis we can find a subsequence $(z_{k_j})_j$ of $(z_k)_k$ which is interpolating for $Hv(G)$. Since $(z_{k_j})_j$ is interpolating, there exists $g \in Hv(G)$ with $g(z_{k_j}) = f_{k_j}(z_{k_j})$ for each $j \in \mathbb{N}$. By assumption g belongs to $Hw_0(G)$; thus, $\lim_{j\to\infty} w(z_{k_j})|g(z_{k_j})| = 0$ which implies

$$\lim_{j\to\infty} w(z_{k_j})|f_{k_j}(z_{k_j})| = 0,$$

a contradiction. □

4. Proposition.

Let v be a weight on the unit disc $\mathbb{D}$ with $Hv(\mathbb{D}) \neq \{0\}$.

(a) Every discrete sequence in $\mathbb{D}$ contains an interpolating sequence for $H\tilde{v}(\mathbb{D})$, where $\tilde{v}$ is the associated weight.

(b) Note that $Hv(\mathbb{D}) = H\tilde{v}(\mathbb{D})$ holds isometrically. Hence,

$$\text{If } Hv(\mathbb{D}) \subset Hw_0(\mathbb{D}) \text{ for some other weight } w \text{ on } \mathbb{D},$$

$$\Downarrow \quad \text{(by Proposition 3)}$$

$$\text{the inclusion map } Hv(\mathbb{D}) \longrightarrow Hw_0(\mathbb{D}) \text{ is compact}$$

Proof. The condition $Hv(\mathbb{D}) \neq \{0\}$ implies that $\tilde{v}$ is a weight; i.e., that $\tilde{v}(z) < +\infty$ for each $z \in \mathbb{D}$. Let $(z_k)_k$ be a discrete sequence in $\mathbb{D}$. We can choose a subsequence $(z_j)_j = (z_{k_j})_j$ of $(z_k)_k$ which is interpolating for $H^\infty(\mathbb{D})$ and a sequence $(\varphi_j)_j \subset H^\infty(\mathbb{D})$ such that $\varphi_j(z_i) = \delta_{ij}$ for all i and j and such that $\sum_j |\varphi_j| \leq M$ on $\mathbb{D}$ for some constant $M > 0$.

Fix $(\alpha_j)_j \subset \mathbb{C}$ with $\sup_j \tilde{v}(z_j)|\alpha_j| =: m < +\infty$. For each $j \in \mathbb{N}$ one can find $f_j \in B_v$ = the unit ball of $Hv(\mathbb{D})$ with $f_j(z_j) = 1/\tilde{v}(z_j)$; cf. [10], 1.2.(iv). Now put $f := \sum_{j=1}^\infty \tilde{v}(z_j)\alpha_j\varphi_j f_j$. We will check that $f \in Hv(\mathbb{D}) = H\tilde{v}(\mathbb{D})$; clearly $f(z_j) = \alpha_j$ for each $j \in \mathbb{N}$. But for any $z \in \mathbb{D}$ we see that

$$\begin{aligned} v(z)|f(z)| &\leq \\ &\leq v(z)\sum_{j=1}^\infty \tilde{v}(z_j)|\alpha_j||\varphi_j(z)||f_j(z)| \\ &\leq M(\sup_j \tilde{v}(z_j)|\alpha_j|)\left(\sup_j \sup_{z\in\mathbb{D}} v(z)|f_j(z)|\right) \\ &\leq Mm. \end{aligned}$$

Since the convergence of the series is uniform on compact subsets of $\mathbb{D}$, f is holomorphic on $\mathbb{D}$. □

5. Proposition.

Let G be an open connected subset of $\mathbb{C}$ such that,

for the Riemann sphere $\mathbb{C}^*$, $\mathbb{C}^* \setminus G$ does not have a connected component consisting of only one point	+	$Hv(G) \neq \{0\}$

$$\Downarrow$$

every discrete sequence in G contains a subsequence which is interpolating for $H\tilde{v}(G)$

where $\tilde{v}$ again denotes the associated weight.

• In particular, this result applies to

$$G = U := \{z \in \mathbb{C};\ \mathrm{Im} z > 0\},$$

the upper half plane.

Let Φ be a (nonharmonic) subharmonic function on $\mathbb{C}$ whose Laplacian $\mu = \Delta\Phi$ (a nonnegative Borel measure, finite on compact sets) is a **doubling measure**; i.e., there is $C > 0$ so that

$$\mu(D(z, 2r)) \leq C\mu(D(z, r)) \quad \forall z \in \mathbb{C}, \quad \forall r > 0,$$

Examples:

(i) The functions

$$\Phi(z) = |z|^\beta, \qquad \Phi(z) = |z|^\beta (\log(1 + |z|^2))^\alpha,$$
$$(\alpha \geq 0, \beta > 0)$$

satisfy this assumption,

(ii) $\Phi(z) = \exp|z|$ does not.

• For each $z \in \mathbb{C}$ let $\rho(z)$ denote the positive radius with

$$\mu(D(z, \rho(z))) = 1.$$

• A sequence $\Lambda \subset \mathbb{C}$ is ρ-**separated** if $\exists\, \delta > 0$ such that

$$|\lambda - \lambda'| \geq \delta \,\text{máx}(\rho(\lambda), \rho(\lambda')) \quad \forall\, \lambda, \lambda' \in \Lambda, \quad \lambda \neq \lambda'.$$

• The **upper uniform density** of Λ with respect to $\mu = \Delta\Phi$ is

$$D^{+}_{\Delta\Phi} := \limsup_{r\to\infty} \sup_{z\in\mathbb{C}} \frac{\#(\Lambda \cap \overline{D(z, r\rho(z))})}{\mu(D(z, r\rho(z)))}$$

For Φ as above we now define the weight v_Φ by

$$v_\Phi(z) := \exp(-\Phi(z)), \quad z \in \mathbb{C}.$$

Marco, Massaneda and Ortega-Cerdà:

The sequence Λ is interpolating for $Hv_\Phi(\mathbb{C})$

$$\Updownarrow$$

Λ *is ρ-separated* *and* $D^{+}_{\Delta\Phi}(\Lambda) < 1/2\pi$

6. Proposition.

Let Φ be a subharmonic function on $\mathbb{C}$ whose Laplacian $\mu = \Delta\Phi$ is a doubling measure.

Every discrete sequence in $\mathbb{C}$ has a subsequence which is interpolating for $Hv_\Phi(\mathbb{C})$.

7. Corollary.

Let v, w be weights on $\mathbb{C}$ with $Hv(\mathbb{C}) \subset Hw_0(\mathbb{C})$.

If the weight v equals $\exp(-\Phi)$ for a subharmonic function Φ such that $\Delta\Phi$ is a doubling measure

$$\Downarrow$$

the inclusion map $Hv(\mathbb{C}) \to Hw_0(\mathbb{C})$ is compact

8. Lemma.

Let G be an absolutely convex open subset of $\mathbb{C}^N$, $N \geq 1$, and $z_0 \in \partial G$. Then there is a linear form w on $\mathbb{C}^N$ such that $w(G) = \mathbb{D}$ and $w(z_0) = 1$.

9. Proposition.

Let G be an absolutely convex open and bounded subset of $\mathbb{C}^N$ and let v be a weight on G such that the associated weight $\tilde{v}$ satisfies $\tilde{v}(z) < +\infty$ for each $z \in G$ (which holds, in particular, if v is bounded). Then every discrete sequence in G contains a subsequence which is interpolating for $H\tilde{v}(G)$.

WEIGHTED INDUCTIVE LIMITS

Let $\mathcal{V} = (v_n)_n$ be a decreasing sequence of weights on G.

The **weighted inductive limits of spaces of holomorphic functions on** G are defined by

$$\mathcal{V}H(G) := \operatorname*{ind}_n Hv_n(G)$$

$$\mathcal{V}_0H(G) := \operatorname*{ind}_n H(v_n)_0(G)$$

- $\mathcal{V}_0H(G)$ is continuously embedded in $\mathcal{V}H(G)$

- It is not clear a priori if it is even a topological subspace

• **Associated system**

$$\overline{V} = \overline{V}(\mathcal{V}) := \{\overline{v} \text{ weight on } G;\ \forall n:\ \sup_G \frac{\overline{v}}{v_n} < +\infty\}$$

• **Projective hulls of the weighted inductive limits**

$H\overline{V}(G) := \{f \in H(G);\ \forall \overline{v} \in \overline{V}:\ p_{\overline{v}}(f) < +\infty\}$,
where

$$p_{\overline{v}}(f) := \sup_G \overline{v}|f|$$

$H\overline{V}_0(G) := \{f \in H(G);\ \overline{v}f \text{ vanishes at } \infty \text{ on } G\ \forall \overline{v} \in \overline{V}\}$

endowed with the topology given by the system

$$\{p_{\overline{v}};\ \overline{v} \in \overline{V}\}$$

of seminorms.

• $H\overline{V}_0(G)$ is a closed topological subspace of $H\overline{V}(G)$.

We have

$$\begin{array}{ccc} \mathcal{V}_0 H(G) & \hookrightarrow & \mathcal{V}H(G) \\ \cap & & \parallel \\ H\overline{V}_0(G) & \hookrightarrow & H\overline{V}(G) \end{array}$$

• Bierstedt, Meise and Summers:

$\mathcal{V}H(G) = H\overline{V}(G)$ and $\mathcal{V}H(G)$ is complete

• If the sequence $\mathcal{V} = (v_n)_n$ is regularly decreasing,

$$\Downarrow$$

$\mathcal{V}_0 H(G) = H\overline{V}_0(G)$ and $\mathcal{V}_0 H(G)$ is complete.

Let $\mathcal{V} = (v_n)_n$ be a decreasing sequence of weights on G.

We denote
$B_n \rightsquigarrow$ the closed unit ball of $Hv_n(G)$
$C_n \rightsquigarrow$ the closed unit ball of $H(v_n)_0(G)$

10. Theorem.

We assume that for each $n \in \mathbb{N}$ every discrete sequence in G contains a subsequence which is interpolating for $Hv_n(G)$.

(a) $\boxed{\mathcal{V}H(G) = \mathcal{V}_0 H(G)}$ $\Rightarrow$ *$\mathcal{V}H(G)$ and $\mathcal{V}_0 H(G)$ are (DFS)-spaces*

(b) $\boxed{H\overline{V}(G) = H\overline{V}_0(G)}$ $\Rightarrow$ *$H\overline{V}(G)$ is semi-Montel*

(c) If, in addition, B_n is contained in the co-closure of C_n for each n, then the converse of (a) and (b) is also true.

PROOF. (a) The hypothesis and Grothendieck's factorization theorem imply that for each n there is $m > n$ with $Hv_n(G) \subset H(v_m)_0(G)$. Proposition 3. then yields that the inclusion map is compact. Restricting to $H(v_n)_0(G)$ we get that the inclusion map $H(v_n)_0(G) \to H(v_m)_0(G)$ is compact. Thus, $\mathcal{V}_0 H(G)$ is a (DFS)-space. On the other hand, composing with the inclusion $H(v_m)_0(G) \subset Hv_m(G)$, we obtain that the mapping $Hv_n(G) \to Hv_m(G)$ is compact; thus, also $\mathcal{V}H(G)$ is a (DFS)-space.

(b) Let B be a bounded subset of $H\overline{V}(G)$. Since $H\overline{V}(G)$ and $\mathcal{V}H(G)$ have the same bounded sets and since the inductive limit $\mathcal{V}H(G) = \mathrm{ind}_n\, Hv_n(G)$ is regular, there exists n such that B is contained and bounded in $Hv_n(G)$. Without loss of generality, we may assume that $B \subset B_n$. Since $H\overline{V}(G)$ is the projective limit of the spaces $H\overline{v}(G)$, $\overline{v} \in \overline{V}$ strictly positive and continuous, it is enough to show that B_n is relatively compact in $H\overline{v}(G)$ for each $\overline{v} \in \overline{V}$ strictly positive and continuous. By the hypothesis we have that $Hv_n(G) \subset H\overline{V}(G) = H\overline{V}_0(G) \subset H\overline{v}_0(G)$. Applying Proposition 3. once more, we conclude that the inclusion $Hv_n(G) \to H\overline{v}_0(G)$ is compact. Therefore B_n is relatively compact in $H\overline{v}_0(G) \subset H\overline{v}(G)$, as desired.

(c) Let $\mathcal{V}H(G)$ be a (DFS)-space and fix $f \in \mathcal{V}H(G)$. Without loss of generality we may assume that $f \in B_n$ for some n. By our hypothesis there is a sequence $(f_j)_j \subset C_n$ with $f_j \to f$ in co. Since $\mathcal{V}H(G)$ is a (DFS)-space, co and the norm topology induced by some $Hv_m(G)$, $m > n$, coincide on B_n. Thus we obtain $f_j \to f$ in $Hv_m(G)$. But $f_j \in H(v_m)_0(G)$ for each $j \in \mathbb{N}$, and it follows that $f \in H(v_m)_0(G) \subset \mathcal{V}_0H(G)$, from which we get the conclusion.

Finally let $H\overline{V}(G)$ be a semi-Montel space and fix $f \in H\overline{V}(G)$. By the algebraic equality $H\overline{V}(G) = \mathcal{V}H(G)$ we get $f \in Hv_n(G)$ for some n and may again assume that $f \in B_n$. By hypothesis there is a sequence $(f_j)_j \subset C_n \subset \mathcal{V}_0H(G) \subset H\overline{V}_0(G)$ with $f_j \to f$ in co. Since $H\overline{V}(G)$ is semi-Montel, co and the weighted topology of $H\overline{V}(G)$ coincide on the bounded subset B_n of $H\overline{V}(G)$. Hence $f_j \to f$ in $H\overline{V}(G)$ and $f \in H\overline{V}_0(G)$. $\square$

Let G be an open subset of $\mathbb{C}^N$ and $\mathcal{V} = (v_n)_n$ a decreasing sequence of weights on G such that B_n is contained in the co-closure of C_n for each n.

11. Proposition.

(a) $\boxed{H\overline{V}(G) = H\overline{V}_0(G)} \Leftrightarrow H\overline{V}_0(G)$ *is semireflexive*

(b) $\boxed{\mathcal{V}H(G) = \mathcal{V}_0H(G)} \Leftrightarrow$ *for each* $n \in \mathbb{N}, \ \exists m > n :$ $i_{n,m} : H(v_n)_0(G) \to H(v_m)_0(G)$ *is weakly compact*

12. Theorem.

In addition to the assumptions of this section we suppose that $\tilde{v}_1(z) < +\infty$ *for each* $z \in G$. *Then*

$$H\overline{V}(G) \text{ is semi-Montel}$$

$$\Updownarrow$$

$$\overline{v}/\tilde{v}_n \text{ vanishes at } \infty \text{ on } G$$
$$\text{for each } \overline{v} \in \overline{V} \text{ and each } n \in \mathbb{N}$$

When the inductive limit $\mathcal{V}_0H(G)$ is complete?

Remark.

If the closed unit ball B_n of $Hv_n(G)$ is contained in the co-closure of the unit ball C_n of $H(v_n)_0(G)$ for each $n \in \mathbb{N}$, then

$$\mathcal{V}_0H(G) = H\overline{V}_0(G) \quad \Rightarrow \quad \mathcal{V}_0H(G) \text{ is complete}$$

• $\mathcal{V} = (v_n)_n$ is **regularly decreasing** $\Rightarrow \mathcal{V}_0H(G) = H\overline{V}_0(G)$

The sequence $\mathcal{V} = (v_n)_n$ is **regularly decreasing** if

$$\forall n \quad \exists m > n \quad \forall Y \subset G : \quad \sup_Y \frac{v_m}{v_n} > 0$$

$$\Downarrow$$

$$\sup_Y \frac{v_k}{v_n} > 0 \qquad \forall k \geq m$$

• In this case $\mathcal{V}H(G) = \text{ind}_n\, Hv_n(G)$ is **boundedly retractive**; i.e., for each bounded set $B \subset \mathcal{V}H(G)$ there is n such that B is a bounded subset of $Hv_n(G)$ and such that the topologies of $\mathcal{V}H(G)$ and $Hv_n(G)$ coincide on B.

13. Theorem.

Let $\mathcal{V} = (v_n)_n$ *be a decreasing sequence of weights on* $\mathbb{D}$ *such that* $\mathcal{V}_0 H(\mathbb{D})$ *is a dense topological subspace of* $H\overline{V}_0(\mathbb{D})$.

$\mathcal{V}_0 H(\mathbb{D})$ *is complete* $\quad\Rightarrow\quad$ $\mathcal{V}_0 H(\mathbb{D}) = \operatorname{ind}_n H(v_n)_0(\mathbb{D})$ *is boundedly retractive*

boundedly retractive	$\Leftrightarrow$	sequentially retractive

- $\mathcal{V}_0 H(\mathbb{D})$ is **sequentially retractive** if

  for each $(f_k)_k \subset \mathcal{V}_0 H(\mathbb{D})$ with $f_k \to 0$ in $\mathcal{V}_0 H(\mathbb{D})$

$$\Downarrow$$

$f_k \to 0$ in $H(v_n)_0(\mathbb{D})$ for some n

14. Corollary.

Let $\mathcal{V} = (v_n)_n$ *be a decreasing sequence of radial weights on the unit disc* $\mathbb{D}$ *with*

$$\lim_{r \to 1-} v_n(r) = 0$$

for each n *and such that* $\mathcal{V}_0 H(\mathbb{D})$ *is a topological subspace of* $H\overline{V}_0(\mathbb{D})$. *Then*

$\mathcal{V}_0 H(\mathbb{D})$ *is complete*

$$\Updownarrow$$

$\mathcal{V}_0 H(\mathbb{D}) = H\overline{V}_0(\mathbb{D})$

$$\Updownarrow$$

$\mathcal{V}_0 H(\mathbb{D}) = \operatorname{ind}_n H(v_n)_0(\mathbb{D})$ *is boundedly retractive*

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