Operators of differentiation and integration on weighted Banach spaces of entire functions

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Aim of the talk

Investigate the dynamics of the operators of

Differentiation: $Df := f'$

Integration: $Jf(z) := \int_0^z f(\xi)d\xi, \ z \in \mathbb{C}$

Hardy operator: $Hf(z) := \frac{1}{z}\int_0^z f(\xi)d\xi, \ z \in \mathbb{C}$

on weighted Banach spaces of entire functions.

- D and J are continuous on $(H(\mathbb{C}), co)$, where co denotes the compact-open topology.
- $DJf = f$ and $JDf(z) = f(z) - f(0) \ \forall f \in H(\mathbb{C}), \ z \in \mathbb{C}$.

Notation

Weights

A *weight* v on $\mathbb{C}$ is a strictly positive continuous function on $\mathbb{C}$ which is radial, i.e. $v(z) = v(|z|)$, $z \in \mathbb{C}$, $v(r)$ is non-increasing on $[0, \infty[$ and rapidly decreasing, that is, it satisfies $\lim_{r\to\infty} r^n v(r) = 0$ for each $n \in \mathbb{N}$.

For $r \geq 0$ and $f \in \mathcal{H}(\mathbb{C})$, consider

$$M_p(f,r) := \left(\frac{1}{2\pi}\int_0^{2\pi} |f(re^{it})|^p dt\right)^{1/p} \quad \text{for } 1 \leq p < \infty$$

and

$$M_\infty(f,r) := \sup_{|z|=r} |f(z)|, \;\; r \geq 0.$$

Note that for each $1 \leq p < \infty$ and each $n \in \mathbb{N}$, we have $M_p(z^n, r) = M_\infty(z^n, r)$ for each $r > 0$.

For a weight v and $1 \leq p \leq \infty$, set

$$B_{p,\infty}(v) := \{f \in H(\mathbb{C}) : \sup_{r>0} v(r) M_p(f,r) < \infty\}$$

and

$$B_{p,0}(v) := \{f \in H(\mathbb{C}) : \lim_{r\to\infty} v(r) M_p(f,r) = 0\}.$$

Both are Banach spaces with the norm

$$|||f|||_{p,v} := \sup_{r>0} v(r) M_p(f,r).$$

In case $p = \infty$, these spaces are usually denoted by $H_v^\infty(\mathbb{C})$ and $H_v^0(\mathbb{C})$, respectively.
We have

$$B_{p,0}(v) \subseteq B_{p,\infty}(v) \subseteq B_{1,\infty}(v) \subseteq H(\mathbb{C})$$

with continuous inclusions for every $1 \leq p \leq \infty$.

Results of Lusky

Structure of the spaces

- The polynomials are included in $B_{p,0}(v)$ for all $1 \leq p \leq \infty$ and they are even dense. In particular, $B_{p,0}(v)$ is separable.
- For $1 < p < \infty$, the monomials are a Schauder basis of $B_{p,0}(v)$, but this is not satisfied in general for $p \in \{1, \infty\}$.
- For every $1 \leq p \leq \infty$ the bidual of $B_{p,0}(v)$ is isometrically isomorphic to $B_{p,\infty}(v)$.

Results of Lusky

Structure of the spaces

- The space $H_v^\infty(\mathbb{C}) = B_{\infty,\infty}(v)$ is isomorphic either to H^∞ or to ℓ_∞. The characterization is in terms of a technical condition on the weight v.

- The space $H_v^0(\mathbb{C}) = B_{\infty,0}(v)$ has a basis.

Examples

Weighted spaces for exponential weights

- Let $1 \leq p \leq \infty$. The space $B_{p,q}(a, \alpha)$, $q = 0$ or $q = \infty$, denotes the Bergman space associated to the following weight: $v_{a,\alpha}(r) = e^{-\alpha}$, $r \in [0, 1[$, $v_{a,\alpha}(r) = r^a e^{-\alpha r}$, $r \geq 1$, if $a < 0$ and $v_{a,\alpha}(r) = (a/\alpha)^a e^{-a}$, $r \in [0, a/\alpha[$, $v_{a,\alpha}(r) = r^a e^{-\alpha r}$, $r \geq a/\alpha$, if $a > 0$.
- In case $a = 0$, $v_{0,\alpha}(r) = e^{-\alpha r}$, and we write $B_{p,q}(\alpha)$.
- The norms will be denoted by $|| \, ||_{p,a,\alpha}$ and $|| \, ||_{p,\alpha}$. If, in addition, $p = \infty$, we simply write $|| \, ||_{a,\alpha}$ and $|| \, ||_{\alpha}$.
- Especially important for us are $H_\alpha^\infty(\mathbb{C}) := B_{\infty,\infty}(\alpha)$ and $H_\alpha^0(\mathbb{C}) := B_{\infty,0}(\alpha)$

The following result is useful in connection with the existence of periodic points for the operators of integration or differentiation.

Proposition (Bonet, Bonilla)

The following conditions are equivalent for a weight v and $1 \leq p \leq \infty$:

(i) $\{e^{\theta z} : |\theta| = 1\} \subset B_{p,0}(v)$.

(ii) There is $\theta \in \mathbb{C}, |\theta| = 1$, such that $e^{\theta z} \in B_{p,0}(v)$.

(iii) $\lim_{r\to\infty} v(r)\frac{e^r}{r^{\frac{1}{2p}}} = 0$.

Proof.

We consider $f(z) = e^z, z \in \mathbb{C}, z = r(\cos t + i \sin t)$.
Applying the Laplace methods for integrals, for $r > 0$,

$$2\pi M_p(f,r)^p = \int_0^{2\pi} e^{rp\cos t} dt = \left(\frac{\pi}{2rp}\right)^{1/2} e^{rp} + e^{rp} O\left(\frac{1}{rp}\right).$$

This yields, for a certain constant $c_p > 0$ depending only on p,

$$M_p(f,r) = c_p \frac{e^r}{r^{\frac{1}{2p}}} + e^r O\left(\frac{1}{r^{\frac{1}{p}}}\right).$$

This implies that for each $1 \leq p < \infty$ there are $d_p, D_p > 0$ and $r_0 > 0$ such that for each $|\theta| = 1$ and each $r > r_0$

$$d_p \frac{e^r}{r^{\frac{1}{2p}}} \leq M_p(e^{\theta z}, r) \leq D_p \frac{e^r}{r^{\frac{1}{2p}}} \qquad (1)$$

The equivalence of conditions (i), (ii) and (iii) in the statement follows easily.

For a Banach space X, we write

$$\mathcal{L}(X) := \{T : X \to X \text{ linear and continuous }\}.$$

Given $T \in \mathcal{L}(X)$, the pair (X, T) is a *linear dynamical system*.

Definitions

- Let $x \in X$. The *orbit of x under T* is the set

$$\mathcal{O}rb(x, T) := \{x, Tx, T^2x, ...\} = \{T^n x : n \geq 0\}.$$

- $x \in X$ is a *periodic point* if $\exists n \in \mathbb{N}$ such that $T^n x = x$.

For a Banach space X and $T \in \mathcal{L}(X)$, we say

Definitions

- T *topologically mixing* $\Leftrightarrow \forall U, V \neq \emptyset$ open, $\exists n_0 : T^n U \cap V \neq \emptyset$ $\forall n \geq n_0$.
- T *hypercyclic* $\Leftrightarrow \exists x \in X,\ \mathcal{O}rb(T, x) := \{x, Tx, T^2x, \dots\}$ is dense in $X \Rightarrow X$ separable.

Definition (Godefroy, Shapiro)

T is *chaotic* if

- T has a dense set of periodic points,
- T is hypercyclic.

Dynamics of linear operators

For a Banach space X and $T \in \mathcal{L}(X)$, we define

Definitions

- T *power bounded* $\Leftrightarrow \sup_n \|T^n\| < \infty$
- T *Cesàro power bounded* $\Leftrightarrow \sup_n \|\frac{1}{n}\sum_{k=1}^{n} T^k\| < \infty$
- T *mean ergodic* $\Leftrightarrow$

$$\forall x \in X, \ \exists Px := \lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} T^k x \in X$$

- T *uniformly mean ergodic* $\Leftrightarrow$

$$\left\{ \frac{1}{n}\sum_{k=1}^{n} T^k \right\}_n$$

converges in the operator norm.

Theorem. Mac Lane (1952).

$D : H(\mathbb{C}) \to H(\mathbb{C})$ is hypercyclic, i.e.,

$$\exists f_0 \in H(\mathbb{C}) : \forall f \in H(\mathbb{C}),\ \exists (n_k)_k \subseteq \mathbb{N} \text{ such that}$$

$$f_0^{(n_k)} \to f \text{ uniformly on compact sets.}$$

Proposition.

The integration operator $J : H(\mathbb{C}) \to H(\mathbb{C})$ is not hypercyclic. In fact, for each $f \in H(\mathbb{C})$, the sequence $(J^n f)_n$ converges to 0 in $H(\mathbb{C})$.

$\mathcal{P}$ is the space of polynomials.

Proposition.

Let $T : (\mathcal{H}(\mathbb{C}), \tau_{co}) \to (\mathcal{H}(\mathbb{C}), \tau_{co})$ be a continuous linear operator such that $T(\mathcal{P}) \subset \mathcal{P}$, let v be a weight and $1 \leq p \leq \infty$. The following conditions are equivalent:

(i) $T(B_{p,\infty}(v)) \subset B_{p,\infty}(v)$.

(ii) $T : B_{p,\infty}(v) \to B_{p,\infty}(v)$ is continuous.

(iii) $T(B_{p,0}(v)) \subset B_{p,0}(v)$.

(iv) $T : B_{p,0}(v) \to B_{p,0}(v)$ is continuous.

Moreover, in this case the norm and the spectrum of the operators coincide.

Harutyunyan, Lusky, 2008: The continuity of D and J on $H_v^\infty(\mathbb{C})$ is determined by the growth or decline of $v(r)e^{\alpha r}$ for some $\alpha > 0$ in an interval $[r_0, \infty[$.

Continuity

Proposition.

Let v be a weight function such that $\sup_{r>0} \frac{v(r)}{v(r+1)} < \infty$ and let $1 \leq p \leq \infty$. Then the differentiation operators $D : B_{p,\infty}(v) \to B_{p,\infty}(v)$ and $D : B_{p,0}(v) \to B_{p,0}(v)$ are continuous.

Proposition.

Let v be a weight such that $v(r) = e^{-\alpha r}$ for some $\alpha > 0$ and let $1 \leq p \leq \infty$. The operator J is continuous on $B_{p,\infty}(v)$ and $B_{p,0}(v)$ with $|||J^n|||_{p,v} = 1/\alpha^n$ for each n.

Hypercyclicity and chaos

Proposition.

Assume that the integration operator $J : B_{p,0}(v) \to B_{p,0}(v)$ is continuous for some $1 \leq p \leq \infty$. The operator J is not hypercyclic and it has no periodic points different from 0.

Theorem (Bonet, Bonilla)

Assume that the differentiation operator $D : B_{p,0}(v) \to B_{p,0}(v)$ is continuous for some $1 \leq p \leq \infty$. The following conditions are equivalent:

(i) $D : B_{p,0}(v) \to B_{p,0}(v)$ satisfies the hypercyclicity criterion.
(ii) $D : B_{p,0}(v) \to B_{p,0}(v)$ is hypercyclic.
(iii) $\lim\ \inf_{n \to \infty} \frac{\|z^n\|_{\infty,v}}{n!} = 0$

Proof for $p = \infty$.

(ii) implies (iii). Assume that D is hypercyclic. Then, there is $f \in H_v^0(\mathbb{C})$, such that $(f^{(n)}(0))_n$ is unbounded in $\mathbb{C}$.

Fix $n \in \mathbb{N}$ and apply the Cauchy inequalities to obtain, for each $r > 0$,

$$v(r)\frac{|f^{(n)}(0)|}{n!}r^n \leq v(r)\max_{|z|=r}|f(z)| \leq ||f||_v.$$

This implies

$$\frac{|f^{(n)}(0)|}{n!}\sup_{z\in\mathbb{C}} v(z)|z^n| \leq ||f||_v,$$

which yields $|f^{(n)}(0)|\frac{||z^n||_v}{n!} \leq ||f||_v$ for each $n \in \mathbb{N}$. Since $(f^{(n)}(0))_n$ is unbounded, we conclude $\liminf_{n\to\infty}\frac{||z^n||_v}{n!} = 0$, which is condition (iii).

Proof for $p = \infty$ continued.

HYPERCYCLICITY CRITERION: Suppose that the continuous operator T on a separable Banach space E satisfies that there exist an increasing sequence $(n_k)_k$ of positive integers, two dense subsets V and W of E and a sequence $(S_{n_k})_k$ of maps, not necessarily linear nor continuous, $S_{n_k} : W \to E$, such that:

(1) $(T^{n_k} v)_k$ converges to 0 for each $v \in V$.

(2) $(S_{n_k} w)_k$ converges to 0 for each $w \in W$.

(3) $(T^{n_k} S_{n_k} w)_k$ converges to w for each $w \in W$.

Then T is hypercyclic.

Proof for $p = \infty$ continued.

Now we prove that (iii) implies (i).
Since D is continuous, there is $C \geq 1$ such that $||f^{(j)}||_v \leq C^j ||f||_v$ for each $f \in H_v^0(\mathbb{C})$ and each $j \in \mathbb{N}$.
Set $n_0 = 0$ and use (iii) inductively to find $n_k \in \mathbb{N}$ with $n_{k+1} > n_k + k + 1$ and

$$\frac{||z^{n_k+k+1}||_v}{(n_k + k + 1)!} \leq \frac{1}{kC^k}.$$

This is the increasing sequence of natural numbers required in the hypercyclicity criterion.

Take $V = W$ as the set of all polynomials and define $S_{n_k} := S^{n_k}$ on W, with S the integration map defined on the monomials by $S(z^n) := \frac{z^{n+1}}{n+1}$.

Proof for $p = \infty$ continued.

Conditions (1) and (3) in the criterium are clear.

We have to show that $\lim_{k\to\infty} S^{n_k} w = 0$ in Hv_0 for each polynomial w. To see this, fix $s \in \mathbb{N} \cup \{0\}$ and take $k \geq s$.
Observe that

$$S^{n_k}(z^s) = \frac{s!}{(n_k + s)!} z^{n_k+s}$$

and

$$D^{k+1-s}(z^{n_k+k+1}) = \frac{(n_k + k + 1)!}{(n_k + s)!} z^{n_k+s}.$$

This implies
We have

$$||S^{n_k}(z^s)||_v = \frac{s!}{(n_k + s)!} ||z^{n_k+s}||_v =$$

$$\frac{s!}{(n_k + k + 1)!} ||D^{k+1-s}(z^{n_k+k+1})||_v \leq s! C^{k+1-s} \frac{||z^{n_k+k+1}||_v}{(n_k + k + 1)!} < s! \frac{1}{k},$$

and the proof is complete.

Hypercyclicity and chaos

Theorem (Bonet, Bonilla)

Assume that the differentiation operator $D : B_{p,0} \to B_{p,0}$ is continuous for some $1 \leq p \leq \infty$. The following conditions are equivalent:

(i) $D : B_{p,0}(v) \to B_{p,0}(v)$ is mixing.

(ii) $\lim_{n\to\infty} \frac{\|z^n\|_{\infty,v}}{n!} = 0$.

Theorem (Bonet, Bonilla)

Let v be a weight function such that the differentiation operator $D : B_{p,0} \to B_{p,0}$ is continuous for some $1 \leq p \leq \infty$. The following conditions are equivalent:

(i) $D : B_{p,0}(v) \to B_{p,0}(v)$ is chaotic.

(ii) $D : B_{p,0}(v) \to B_{p,0}(v)$ has a periodic point different from 0.

(iii) $\lim_{r\to\infty} v(r)\frac{e^r}{r^{\frac{1}{2p}}} = 0$.

Hypercyclicity and chaos. An example

Corollary.

The operator $D : B_{\infty,0}(a,\alpha) \to B_{\infty,0}(a,\alpha)$ satisfies

- $0 < \alpha < 1 \implies D$ is not hypercyclic and has no periodic point different from 0.
- $\alpha = 1 \implies$ if $a < 1/2$, then D is topologically mixing, and if $a \geq 1/2$, D is not hypercyclic. It has no periodic point different from 0 iff $a \geq 0$.
- $\alpha > 1 \implies D$ is chaotic and topologically mixing.

Norms, spectrum and mean ergodicity

From now on, to simplify the notation and the exposition, we will concentrate on the operators D and J defined on the spaces

$$H_v^\infty(\mathbb{C}) = B_{\infty,\infty}(v) \text{ and } H_v^0(\mathbb{C}) = B_{\infty,0}(v).$$

More general results are available, but will not be mentioned in the lecture.

If $v(r) = r^a e^{-\alpha r}$ $(\alpha > 0,\ a \in \mathbb{R})$ for $r \geq r_0$: $||z^n||_{a,\alpha} \approx \left(\frac{n+a}{e\alpha}\right)^{n+a}$, with equality for $a = 0$.

Proposition.

For $a > 0$:

$$||D^n||_{a,\alpha} = \mathcal{O}\left(n! \left(\frac{e\alpha}{n-a}\right)^{n-a}\right) \text{ and } n! \left(\frac{e\alpha}{n+a}\right)^{n+a} = \mathcal{O}(||D^n||_{a,\alpha}).$$

For $a \leq 0$:

$$||D^n||_{a,\alpha} \approx n! \left(\frac{e\alpha}{n+a}\right)^{n+a}$$

and the equality holds for $a = 0$.

Proposition.

For every $\alpha > 0$ and $a \in \mathbb{R}$, the spectrum $\sigma_{a,\alpha}(D) = \alpha\overline{\mathbb{D}}$.

Proposition.

Let v be a weight such that D is continuous on $H_v^\infty(\mathbb{C})$ and that $v(r)e^{\alpha r}$ is non increasing for some $\alpha > 0$. If $|\lambda| < \alpha$, the operator $D - \lambda I$ is surjective on $H_v^\infty(\mathbb{C})$ and on $H_v^0(\mathbb{C})$ and it even has a continuous linear right inverse

$$K_\lambda f(z) := e^{\lambda z} \int_0^z e^{-\lambda \xi} f(\xi) d\xi, \ z \in \mathbb{C}.$$

This was proved by Atzmon, Brive (2006) for $a = 0$.

Norms, spectrum. Integration operator

Proposition.

For the weight $v(r) = r^a e^{-\alpha r}$ ($\alpha > 0$, $a \in \mathbb{R}$) for r big enough, we have

- $||J^n||_{a,\alpha} \cong 1/\alpha^n$, with the equality for $a = 0$.
- $\sigma_{a,\alpha}(J) = (1/\alpha)\overline{\mathbb{D}}$.
- $J - \lambda I$ is not surjective on $B_{\infty,\infty}(a,\alpha)$ or $B_{\infty,0}(a,\alpha)$ if $|\lambda| \leq 1/\alpha$.

Norms and spectrum. Hardy operator

Theorem

For v an arbitrary weight, the Hardy operator $H : H_v^\infty(\mathbb{C}) \to H_v^\infty(\mathbb{C})$ is continuous with norm $\|H\|_v = 1$. Moreover, $H^2(H_v^\infty(\mathbb{C})) \subset H_v^0(\mathbb{C})$ and H^2 is compact. Therefore, $\sigma(H) = \{\frac{1}{n}\}_{\mathbb{N}} \cup \{0\}$. If the integration operator $J : H_v^\infty(\mathbb{C}) \to H_v^\infty(\mathbb{C})$ is continuous, then H is compact.

The operator H is power bounded and uniformly mean ergodic on $H_v^\infty(\mathbb{C})$ and not hypercyclic on $H_v^0(\mathbb{C})$

Remark

For the weight $v(r) = \exp(-(\log r)^2)$:

- J is not continuous on $H_v^\infty(\mathbb{C})$ (Harutyunyan, Lusky)
- $H : H_v^\infty(\mathbb{C}) \to H_v^0(\mathbb{C})$, $H : H_v^0(\mathbb{C}) \to H_v^0(\mathbb{C})$, are compact (Lusky).

Mean ergodicity

Proposition.

Let $T = D$ or $T = J$ and assume $T : H_v^\infty(\mathbb{C}) \to H_v^\infty(\mathbb{C})$ is continuous. The following conditions are equivalent:

(i) $T : H_v^\infty(\mathbb{C}) \to H_v^\infty(\mathbb{C})$ is uniformly mean ergodic.

(ii) $T : H_v^0(\mathbb{C}) \to H_v^0(\mathbb{C})$ is uniformly mean ergodic.

(iii) $\lim_{N\to\infty} \frac{||T+\cdots+T^N||_v}{N} = 0$.

Moreover, if $1 \in \sigma_v(T)$, then T is not uniformly mean ergodic.

Mean ergodicity. Two useful results.

Theorem (Lin)

Let $T \in \mathcal{L}(X)$ such that $\|T^n/n\| \to 0$. Then,

$$T \text{ uniformly mean ergodic } \iff (I - T)X \text{ is closed }.$$

Theorem (Lotz)

Let $T \in \mathcal{L}(H_\alpha^\infty)$ such that $\|T^n/n\| \to 0$. Then,

$$T \text{ mean ergodic } \iff T \text{ uniformly mean ergodic }.$$

H_α^∞ is a Grothendieck Banach space with the Dunford-Pettis property, since it is isomorphic to ℓ_∞ by a result due to Galbis.

Examples

Recall

$$f \in H_\alpha^\infty(\mathbb{C}) \iff \sup_{z \in \mathbb{C}} |f(z)| \exp(-\alpha|z|) < \infty$$

and

$$f \in H_\alpha^0(\mathbb{C}) \iff \lim_{|z| \to \infty} |f(z)| \exp(-\alpha|z|) = 0.$$

Mean ergodicity of the differentiation operator.

Theorem.

Let $v(r) = e^{-\alpha r}$, $r \geq 0$.

- D is power bounded on $H_\alpha^\infty(\mathbb{C})$ or $H_\alpha^0(\mathbb{C})$ if and only if $\alpha < 1$.
- D is uniformly mean ergodic on $H_\alpha^\infty(\mathbb{C})$ and $H_\alpha^0(\mathbb{C})$ if $\alpha < 1$.
- D not mean ergodic if $\alpha > 1$, and
- D is not mean ergodic on $H_1^\infty(\mathbb{C})$ and not uniformly mean ergodic on $H_1^0(\mathbb{C})$.

Mean ergodicity of the integration operator

Theorem.

Let $v(r) = e^{-\alpha r}$, $r \geq 0$.

- J is never hypercyclic.
- J is power bounded on $H_\alpha^\infty(\mathbb{C})$ or $H_\alpha^0(\mathbb{C})$ if and only if $\alpha \geq 1$.
- If $\alpha > 1$, J is uniformly mean ergodic on $H_\alpha^\infty(\mathbb{C})$ and $H_\alpha^0(\mathbb{C})$.
- J is not mean ergodic on these spaces if $\alpha < 1$.
- If $\alpha = 1$, then J is not mean ergodic on $H_1^\infty(\mathbb{C})$, and mean ergodic but not uniformly mean ergodic on $H_1^0(\mathbb{C})$.

Summary

J	$0 < \alpha < 1$	$\alpha = 1$	$\alpha > 1$
Power bounded	no	yes	yes
Hypercyclic on H_α^0	no	no	no
Mean ergodic on H_α^0	no	yes	yes
Mean ergodic on H_α^∞	no	no	yes
Uniformly mean ergodic	no	no	yes

D	$0 < \alpha < 1$	$\alpha = 1$	$\alpha > 1$
Power bounded	yes	no	no
Hypercyclic on H_α^0	no	yes	yes
Top. mixing on H_α^0	no	yes	yes
Chaotic on H_α^0	no	no	yes
Mean ergodic on H_α^0	yes	?	no
Mean ergodic on H_α^∞	yes	no	no
Uniformly mean ergodic	yes	no	no

Open problem

(1) Is the operator of differentiation D mean ergodic on $H_1^0(\mathbb{C})$?

In other words:

Assume that $f \in H(\mathbb{C})$ satisfies $\lim_{|z|\to\infty} |f(z)| \exp(-|z|) = 0$.
Does it follow that

$$\lim_{n\to\infty} \frac{1}{n} \sup_{z\in\mathbb{C}} |f'(z) + \cdots + f^{(n)}(z)| \exp(-|z|) = 0?$$

A related abstract result (A. Peris, 2013)

(2) Are there mean ergodic operators on a separable Banach space that are hypercyclic?

A. Peris has constructed hypercylcic (even mixing) operators on a Banach space that are even uniformly mean ergodic. The example is a variation of a recent example due to F. Martínez, Oprocha and him, that appeared in 2013 in Math. Z.

It is clear that no power bounded operator can be hypercyclic. There are examples classical of mean ergodic operators T on a Banach space such that the sequence $(||T^n||)_n$ tends to infinity due to Hille in 1945. A general construction was presented by Tomilov and Zemanek in 2004.

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