Solid hulls and cores of weighted H^∞-spaces

José Bonet (IUMPA, UPV)

Madrid, February 2018

Joint work with W. Lusky and J. Taskinen

UNIVERSIDAD POLITECNICA DE VALENCIA

IUMPA
Instituto Universitario de Matemática Pura y Aplicada

Aim of the lecture

AIM

Determine the solid hull and solid core of weighted Banach spaces H_v^∞ of analytic functions functions f such that $v|f|$ is bounded, both in the case of the holomorphic functions on the disc and on the whole complex plane, for a very general class of strictly positive, continuous, radial weights v.

We report on joint work with W. Lusky (Univ. Paderborn, Germany) and J. Taskinen (Univ. Helsinki, Finland).

Wolfgang Lusky (Paderborn)

Jari Taskinen (Helsinki)

Solid spaces

An holomorphic function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ on the unit disc $\mathbb{D}$ or on the complex plane $\mathbb{C}$ is identified with the sequence of its Taylor coefficients $(a_n)_{n=0}^{\infty}$, that is also denoted sometimes by (a_n).

Solid spaces

Let A and B be vector spaces of complex sequences containing the space of all the sequences with finitely many non-zero coordinates. The space A is **solid** if $a = (a_n) \in A$ and $|b_n| \leq |a_n|$ for each n implies $b = (b_n) \in A$.

Solid spaces

The **solid hull of** A is

$$S(A) := \{(c_n) \,:\, \exists (a_n) \in A \text{ such that } |c_n| \le |a_n| \ \forall n \in \mathbb{N}\}.$$

The **solid core of** A is

$$s(A) := \{(c_n) \,:\, (c_n a_n) \in A \ \forall (a_n) \in \ell_\infty\}.$$

The **set of multipliers form** A **into** B is

$$(A, B) := \{c = (c_n) \,:\, (c_n a_n) \in B \ \forall (a_n) \in A\}.$$

Solid spaces

Facts: 1. A is solid if and only if $\ell_\infty \subset (A, A)$.

2. $A \subset (B, C)$ if and only if $B \subset (A, C)$.

3. The solid core $s(A)$ of A is the largest solid space contained in A. Moreover $s(A) = (\ell_\infty, A)$.

4. The solid hull $S(A)$ of A is the smallest solid space containing A.

5. If X is solid, $(A, X) = (S(A), X)$ and $(X, A) = (X, s(A))$.

Preliminary examples. Non-Banach spaces

The spaces Fréchet $H(\mathbb{D})$ and $H(\mathbb{C})$ of analytic functions on the unit disc and on the whole complex plane are solid.

Set $R = 1$ (for the case of holomorphic functions on the unit disc) and $R = +\infty$ (for the case of entire functions).
Put $G_R := \{z \in \mathbb{C}\ 1\ |z| < R\}$ and fix a strictly increasing sequence $(r_k)_k \subset]0, R[$ tending to R as k goes to ∞.

It follows from the Cauchy integral formula that

The function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ belongs to $H(G_R)$ if and only if $\sum_{n=0}^{\infty} |a_n|(r_k)^n < \infty$ for each k.

This implies that $H(G_R)$ is solid.

Preliminary examples. Non-Banach spaces

The **Korenblum space** $A^{-\infty}$ of analytic functions on the unit disc is the space of all the functions $f \in H(\mathbb{D})$ such that there is $k \in \mathbb{N}$ such that $\sup_{z \in \mathbb{D}} |f(z)|(1-|z|)^k < \infty$.

It follows again from the Cauchy integral formula that

$f(z) = \sum_{n=0}^{\infty} a_n z^n \in A^{-\infty}$ if and only if there is $k \in \mathbb{N}$ such that $\sup_{n=0,1,2,\dots} |a_n| n^{-k} < \infty$.

This implies that $A^{-\infty}$ is solid.

Hardy spaces

- The concept of solid hulls and solid cores in the context of spaces of holomorphic functions was introduced by **Anderson and Shields** in 1976.

- The solid hull of the Hardy spaces $S(H^p) = H^2, 2 \leq p \leq \infty$ is known.

  The proof for H^∞ depends on a very deep result of **Kislyakov** from 1981.

  The solid hull $S(H^p)$ for $1 \leq p < 2$ seems to be unknown.

Hardy spaces

- The solid core of the Hardy spaces $s(H^p) = H^2, 1 \leq p \leq 2$, is known. Moreover $s(H^\infty) = \ell_1$.

  It is an open problem to describe the solid core $s(H^p)$ for $2 < p < \infty$.

- The space H^2 is solid (with our identification it is well-known that it coincides with ℓ_2) and the space H^∞ is not solid.

H^∞ is not solid. Direct argument

- $T : \ell_1 \to H^\infty$, $T(a_n) := \sum_{n=0}^\infty a_n z^n$ is well-defined, injective and continuous with norm 1.
- If all $f(z) = \sum_{n=0}^\infty a_n z^n \in H^\infty$ satisfies $\sum_{n=0}^\infty |a_n| < \infty$, then T is surjective, hence an isomorphism. Impossible: H^∞ would be separable.
- There is $g(z) = \sum_{n=0}^\infty a_n z^n \in H^\infty$ with $\sum_{n=0}^\infty |a_n| = \infty$.
- Then $\sum_{n=0}^\infty |a_n| z^n \notin H^\infty$, since $b_n \geq 0$ for all n and $\sup_{0<r<1} \sum |b_n| r^n < \infty$ implies $\sum |b_n| < \infty$.
- H^∞ is not solid. The same argument proves $s(H^\infty) = \ell_1$.

If $f(z) = \sum_{n=0}^{\infty} a_n z^n \in H^\infty$, then for each $0 < r < 1$,

$$\sum_{n=0}^{\infty} |a_n|^2 r^{2n} = \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^2 d\theta \leq (||f||_\infty)^2.$$

This implies $(a_n) \in \ell_2$. Therefore $S(H^\infty) \subset \ell_2$.

The inclusion $\ell_2 \subset S(H^\infty)$ depends on the following deep result

Theorem. Kislyakov

There is $B > 0$ such that for each $(b_j)_{j=n}^m, 0 < n < m$, there is a polynomial $P(z) = \sum_{j=n}^m c_j z^j$ such that $|b_j| \leq |c_j|, j = n, ..., m$ and $||P||_\infty \leq B(\sum_{j=n}^m |b_j|^2)^{1/2}$.

Our context

We set $R = 1$ for the case of holomorphic functions on the unit disc and $R = +\infty$ for the case of entire functions.
A **weight** v is a continuous function $v : [0, R[\to]0, \infty[$, which is non-increasing on $[0, R[$ and satisfies $\lim_{r\to R} r^n v(r) = 0$ for each $n \in \mathbb{N}$.
We extend v to $\mathbb{D}$ if $R = 1$ and to $\mathbb{C}$ if $R = +\infty$ by $v(z) := v(|z|)$.

The **weighted Banach spaces of holomorphic functions on** $G = \mathbb{D}$ **or** $G = \mathbb{C}$ are defined by

$$H_v^\infty := \{f \in H(G) \mid \|f\|_v := \sup_{z\in\mathbb{C}} v(|z|)|f(z)| < \infty\},$$
$$H_v^0 := \{f \in H(G) \mid \lim_{|z|\to\infty} v(|z|)|f(z)| = 0\},$$

and they are endowed with the weighted sup norm $\|\cdot\|_v$.

Definitions

- H_v^0 is a closed subspace of H_v^∞.
- The polynomials are contained and dense in H_v^0 but the monomials do not in general form a Schauder basis (**Lusky**).
- The Cesàro means of the Taylor polynomials satisfy $\|C_n f\|_v \leq \|f\|_v$ for each $f \in H_v^\infty$ and the sequence $(C_n f)_n$ is $\|\cdot\|_v$-convergent to f when $f \in H_v^0$.
- Spaces of type H_v^∞ and H_v^0 appear in the study of growth conditions of analytic functions and have been investigated by many authors since the work of Shields and Williams in 1971. Classical operators on these spaces have been also investigated thoroughly.

Examples of weights

For $R = 1$:

(i) $v(r) = (1-r)^{\alpha}$ with $\alpha > 0$, which are the **standard weights** on the disc,

(ii) $v(r) = \exp(-(1-r)^{-1})$.

(iii) $v(z) = (\log \frac{e}{1-r})^{-\alpha}, \ \alpha > 0$

For $R = +\infty$:

(i) $v(r) = \exp(-r^{p})$ with $p > 0$,

(ii) $v(r) = \exp(-\exp r)$,

(iii) $v(r) = \exp\big(-(\log^{+} r)^{p}\big)$, where $p \geq 2$ and $\log^{+} r = \max(\log r, 0)$.

Previous work

- **Bennet, Stegenga and Timoney** determined in 1981 the solid hull and the solid core of the weighted spaces $H_v^\infty(\mathbb{D})$ for doubling weights v on the open unit disc $\mathbb{D}$.
- The weight v on $\mathbb{D}$ is **doubling** if there is $M > 0$ such that $v(1-r) \leq Mv(1-(r/2))$ for each $0 < r < 1$.
- The standard weights are doubling, but the exponential weights $v(r) = \exp(-a/(1-r)^b)$ with $a, b > 0$ are not.

Previous work

- The solid hull and multipliers on spaces of analytic functions on the disc has been investigated by many authors. In addition to those mentioned above, **Anderson, Dostanić, Blasco, Buckley, Jevtić, Pavlović, Ramanujan, Shields and D. Vukotic, among many others**.

- Two recent books about this topic have been recently published by **M. Jevtić, D. Vukotić, M. Arsenović (2016) and by M. Pavlović (2014).**

Previous work

In the case of a standard weight $v_\alpha(z) = (1-|z|^2)^\alpha, \alpha > 0$, we denote $H^\infty_\alpha := H^\infty_{v_\alpha}$.

Theorem.

The solid hull $S(H^\infty_\alpha)$ is

$$S(H^\infty_\alpha) = \left\{ (b_m)_{m=0}^\infty \ : \ \sup_{n\in\mathbb{N}_0} \left(\sum_{m=2^n}^{2^{n+1}-1} |b_m|^2 (m+1)^{-2\alpha} \right)^{1/2} < \infty \right\}.$$

Moreover, the solid core $s(H^\infty_\alpha)$ is

$$s(H^\infty_\alpha) = \left\{ (b_m)_{m=0}^\infty \ : \ \sup_{n\in\mathbb{N}_0} \left(\sum_{m=2^n}^{2^{n+1}-1} |b_m| (m+1)^{-\alpha} \right) < \infty \right\}.$$

These results can be found in the book by M. Jevtić, D. Vukotić, M. Arsenović (2016).

Remark.

$S(H_v^\infty) = H_v^\infty$ if and only if $s(H_v^\infty) = H_v^\infty$.

For a holomorphic function $f(z) = \sum_{n=0}^\infty a_n z^n$, we let h_f denote the function defined by $h_f(z) = \sum_{n=0}^\infty |a_n| z^n$ for all z.

The solid core $s(H_v^\infty)$ of H_v^∞ coincides with the set

$$\{f : f \text{ holomorphic}, h_f \in H_v^\infty\}.$$

Example.

Consider the weight $v(r) = \exp(-\log^2(r))$ on the complex plane $\mathbb{C}$. A result of Lusky (2000) implies that $S(H_v^\infty) = H_v^\infty = s(H_v^\infty)$ and H_v^∞ is solid in this example.

Schauder bases and solid hulls and cores for H_v^∞-spaces

Recall that a sequence $(e_n)_{n=1}^\infty$ of elements of a separable Banach-space X is a **Schauder basis**, if every element $f \in X$ can be presented as a convergent sum

$$f = \sum_{n=1}^{\infty} f_n e_n,$$

where the numbers $f_n \in \mathbb{K}$ are unique for $f \in X$.

Schauder bases and solid hulls and cores. Main results.

We denote by $\Lambda = \{z^k : k = 0, 1, 2, \ldots\}$ the sequence of monomials.

Theorem.

If $S(H_v^\infty) = H_v^\infty$ then Λ is a Schauder basis of H_v^0.

By a Theorem of Lusky (2000), Λ is never a basis for $H_v^0(\mathbb{D})$. This implies the following consequence:

Corollary.

In the case of analytic functions on the disc $\mathbb{D}$, one always has $S(H_v^\infty(\mathbb{D})) \neq H_v^\infty(\mathbb{D})$ and $s(H_v^\infty(\mathbb{D})) \neq H_v^\infty(\mathbb{D})$.

Idea of the proof of the Theorem. For $N \subset \mathbb{N}$, set

$$T_N(\sum_{k=0}^{\infty} a_k z^k) = \sum_{k \in N} a_k z^k.$$

If H_v^∞ is solid, then $T_N(H_v^\infty) \subset H_v^\infty$ and T_N is bounded.

Let P_n be the Dirichlet projections, i.e. $P_n(\sum_{k=0}^{\infty} a_k z^k) = \sum_{k=0}^{n} a_k z^k$.

Assume that Λ is not a basis for H_v^0. By the uniform boundedness theorem there are $f \in H_v^0$ and a subsequence P_{n_m} with

$$\lim_{m \to \infty} \|(P_{n_{m+1}} - P_{n_m})(f)\|_v = \infty.$$

Put $f_m = (P_{n_{m+1}} - P_{n_m})(f)$. Then, $\sum_m f_m \in H_v^\infty$, since this sum is of the form $T_N f$ for some subset N of $\mathbb{N}$.

Idea of the proof of the Theorem continued.

Recall $f_m = (P_{n_{m+1}} - P_{n_m})(f)$.

One can see that there is a subsequence f_{m_k} such that $\sup_k \|f_{m_k}\|_v \leq 2\|\sum_k f_{m_k}\|_v$ (**technical lemma**)

The left hand side of this inequality is infinite while the function on the right-hand side is again of the form $T_{\tilde{N}}f$ and thus has finite norm as an element H_v^∞. A contradiction.

Lusky proved in 2000 that the monomials $\Lambda = \{z^k : k = 0, 1, 2, \ldots\}$ are a basis of H_v^0 for the weight $v(r) = \exp(-\log^2(r))$ on the complex plane $\mathbb{C}$ mentioned before.

In the case of weighted spaces of entire functions we have the following result.

Theorem.

let v be a weight on the complex plane satisfying **condition (b)** (given later). The space H_v^∞ is solid if and only if Λ is a Schauder basis of H_v^0.

Rough idea of the proof of the converse:

If Λ is a Schauder basis of H_v^0, then H_v^∞ is isomorphic to an ℓ_∞-sum of subspaces F_n spanned by the monomials $z^{m_n}, \ldots, z^{m_{n+1}-1}$. **Warning: This requires a delicate argument**.

Since the monomials are a basis and the weighted sup-norm norm on each summand is uniformly isomorphic to the un-weighted sup-norm on the boundary, the dimension of the $F_n's$ must be uniformly bounded and Λ is equivalent to the unit vector basis of c_0. From this one can infer that H_v^∞ is solid.

Examples of solid hulls and cores of H_v^∞

We now know that the weighted Banach spaces H_v^∞ are VERY SELDOM solid.

We still MUST determine the solid hull and solid core of weighted Banach spaces H_v^∞ for cencrete weights on the disc and on the complex plane, as PROMISED.

Solid hull and core. Entire functions.

Theorem.

Let v be the weight $v(r) = \exp(-ar^p)$ on $\mathbb{C}$, where $a > 0$ and $p > 0$ are constants. Then, the solid hull of $H_v^\infty(\mathbb{C})$ is

$$\left\{ (b_m)_{m=0}^\infty \ : \ \sup_{n\in\mathbb{N}} \sum_{pn^2+1<m\le p(n+1)^2} |b_m|^2 e^{-2n^2} n^{4m/p} (ap)^{-m/p} < \infty \right\}.$$

and the solid core is

$$\left\{ (b_m)_{m=0}^\infty \ : \ \sup_{n\in\mathbb{N}} \sum_{pn^2+1<m\le p(n+1)^2} |b_m| e^{-n^2} n^{2m/p} (ap)^{-m/2p} < \infty \right\}.$$

Solid hull and core. Entire functions.

In particular, the solid hull for $v(r) = \exp(-r)$ is

$$\Big\{ (b_m)_{m=0}^{\infty} \; : \; \sup_{n \in \mathbb{N}} \sum_{m=n^2+1}^{(n+1)^2} |b_m|^2 e^{-2n^2} n^{4m} < \infty \Big\}.$$

And the solid core for $v(r) = \exp(-r)$ is

$$\Big\{ (b_m)_{m=0}^{\infty} \; : \; \sup_{n \in \mathbb{N}} \sum_{m=n^2+1}^{(n+1)^2} |b_m| e^{-n^2} n^{2m} < \infty \Big\}.$$

Solid hull. Exponential weights on the disc

Theorem.

For $v(r) = \exp(-1/(1-r))$ the solid hull is

$$\left\{ (b_m)_{m=0}^{\infty} \ : \ \sup_n \exp(-2n^2) \sum_{m=n^4+1}^{(n+1)^4} |b_m|^2 \left(1 - \frac{1}{n^2}\right)^{2m} < \infty \ \right\},$$

and the solid core is

$$\left\{ (b_m)_{m=0}^{\infty} \ : \ \sup_n \exp(-n^2) \sum_{m=n^4+1}^{(n+1)^4} |b_m| \left(1 - \frac{1}{n^2}\right)^{m} < \infty \ \right\},$$

QUESTION

WHERE ALL THESE NUMBERS COME FROM?

HOW DO YOU GET THEM?

A condition on the weight v

Condition (b)

Let $r_m \in]0, R[$ be a global maximum point of the function $r^m v(r)$ for any $m > 0$.

The weight v satisfies the **condition** (b) if there exist numbers $b > 2$, $K > b$ and $0 < m_1 < m_2 < \ldots$ with $\lim_{n \to \infty} m_n = \infty$ such that

$$b \leq \left(\frac{r_{m_n}}{r_{m_{n+1}}} \right)^{m_n} \frac{v(r_{m_n})}{v(r_{m_{n+1}})}, \left(\frac{r_{m_{n+1}}}{r_{m_n}} \right)^{m_{n+1}} \frac{v(r_{m_{n+1}})}{v(r_{m_n})} \leq K.$$

Standard and exponential weights on the disc, and all the examples of weights on the complex plane given above satisfy condition (b).

Solid hull and solid core. Weights with condition (b)

Main General Theorem.

If the weight v satisfies (b), we have

$$S(H_v^\infty) = \Big\{ (b_m)_{m=0}^\infty : \sup_n v(r_{m_n}) \Big(\sum_{m_n < m \leq m_{n+1}} |b_m|^2 r_{m_n}^{2m} \Big)^{1/2} < \infty \Big\},$$

and

$$s(H_v^\infty) = \Big\{ (b_m)_{m=0}^\infty : \sup_n v(r_{m_n}) \Big(\sum_{m_n < m \leq m_{n+1}} |b_m| r_{m_n}^{m} \Big) < \infty \Big\}.$$

The proof is mainly based on results and techniques of Lusky (2006) and methods due to Bennet, Stegenga and Timoney (1981), in particular the theorem of Kislyakov (1981).

An important ingredient in the proof of the main general theorem

Let $h(z) = \sum_{k=0}^{\infty} b_k z^k$.

Put $M_\infty(h, r) = \sup_{|z|=r} |h(z)|$.

For the numbers m_n as in condition (b) set

$$(R_n h)(z) = \sum_{k=0}^{m_{n-1}} b_k z^k + \sum_{m_{n-1} < k \leq m_n} \frac{[m_n] - k}{[m_n] - [m_{n-1}]} b_k z^k.$$

An important ingredient in the proof of the main general theorem

Define the operators $V_n = R_n - R_{n-1}$ for $n \in \mathbb{N}$.

Theorem. Lusky, 2006

The norms

$$||h||_v \quad \text{and} \quad \sup_n \sup_{r_{m_{n-1}} \le r \le r_{m_{n+1}}} M_\infty(V_n h, r) v(r)$$

are equivalent.

Moreover, the operators V_n are uniformly bounded on H_v^∞ and there are $c_1 > 0$ and $c_2 > 0$ with

$$c_1 \sup_n ||V_n h||_v \le ||h||_v \le c_2 ||V_n h||_v \quad \text{for all } h \in H_v^\infty.$$

THE MAIN PROBLEM REMAINS

ONE HAS TO CALCULATE (OR ESTIMATE) THE NUMBERS m_n IN CONDITION (b).

We must guess the value of m_n and then estimate the infimum and supremum of

$$\left(\frac{r_{m_n}}{r_{m_{n+1}}}\right)^{m_n} \frac{v(r_{m_n})}{v(r_{m_{n+1}})} \quad \text{and} \quad \left(\frac{r_{m_{n+1}}}{r_{m_n}}\right)^{m_{n+1}} \frac{v(r_{m_{n+1}})}{v(r_{m_n})}$$

The case of entire functions

Proposition.

Let $v(r) = \exp(-ar^p)$, $r \in [0, \infty[$, $a > 0$, $p > 0$ and let $b > 2$. The weight v satisfies condition (b) for the sequence $m_n := p(\log b)n^2$ with $K = b^5$.

In this case it is possible to make explicit calculations and estimates, because the maximum point r_m of $r^m v(r)$ is $r_m = (m/ap)^{1/p}$, and $v(r_m) = \exp(-m/p)$ for each $m \in \mathbb{N}$.

Exponential weights on the disc

We consider weights of the form

$$v(z) = w(r) \exp\left(-\frac{a}{(1-r)^b}\right), \quad z \in \mathbb{D},$$

where $a, b > 0$ are given constants and $w : [0,1[\to]0,\infty[$ is a differentiable function, extended to $\mathbb{D}$ by $w(z) = w(|z|)$.

This frame includes the exponential weights for $w \equiv 1$.

Exponential weights on the disc

Let $v(z) = w(r) \exp\left(-\frac{a}{(1-r)^b}\right)$.

Assume that $w'(r)/w(r)$ is a decreasing function and assume that there are $n_0 > 0$ and $\alpha \in]0, 1 + b/2[$ such that

$$\sup_{r\in[0,1[} (1-r)^\alpha \frac{w'(r)}{w(r)} < \infty$$

and

$$\frac{1}{e} \le \frac{w(1-(\frac{a}{bn^2})^{1/b})}{w(1-(\frac{a}{b(n+1)^2})^{1/b})} \le e, \quad n \ge n_0.$$

Then, one can take the numbers (m_n) as follows

Exponential weights on the disc

$$m_n = b\left(\frac{b}{a}\right)^{1/b} n^{2+2/b} - bn^2 - \left(1 - \left(\frac{a}{bn^2}\right)^{1/b}\right)\frac{w'(1-\left(\frac{a}{bn^2}\right)^{1/b})}{w(1-\left(\frac{a}{bn^2}\right)^{1/b})}$$

If $w \equiv 1$, this reduces to

$$m_n = b^{1+1/b} a^{-1/b} n^{2+2/b} - bn^2.$$

Observe that in this case one cannot calculate explicitly the maximum point r_m as in the case of entire functions.

Exponential weights on the disc. Examples

(i) If $a = b = 1$, $w = 1$, $v(z) = \exp\left(-\frac{1}{(1-r)}\right)$ and we can take

$$m_n = n^4 - n^2.$$

One can also take in the representation of the solid hull the numbers $\tilde{m}_n = n^4$ instead.

(ii) If $a = 1$, $b = 2$, $w(r) = 1 - r$, $v(z) = (1 - r)\exp\left(-\frac{1}{(1-r)^2}\right)$ and we can take

$$m_n = 2^{3/2}n^3 - 2n^2 + 2^{1/2}n - 1.$$

(iii) $a = b = 1$, $w(r) = (1 - \log(1 - r))^{-1}$,
$v(z) = (1 - \log(1 - r))^{-1} \exp\left(-\frac{1}{(1-r)}\right)$. In this case a direct calculation yields

$$m_n = n^4 - n^2 + \frac{n^2 - 1}{1 + \log(n^2)}.$$

(iv) $a = b = 1$, $w(r) = \exp(-\log^2(1 - r))$,
$v(z) = \exp(-\log^2(1 - r)) \exp\left(-\frac{1}{(1-r)}\right)$. Here we have $w'(r)/w(r) = 2(1 - r)^{-1}\log(1 - r)$ and we obtain

$$m_n = n^4 - n^2 + 4(n^2 - 1)\log(n).$$

Recall that the set of multipliers from A into B is

$$(A, B) := \{c = (c_n) \,:\, (c_n a_n) \in B \ \forall (a_n) \in A\}.$$

Results of Kellogg (1971) and Blasco, Zaragoza-Berzosa (2014) permit us to obtain:

Corollary.

Let $v(r) = e^{-r}$, $r \in (0, \infty)$ and $1 \leq p \leq \infty$. Then, the space of multipliers $\left(H_v^\infty(\mathbb{C}), \ell_p\right)$ is the set of sequences $(\lambda_m)_{m=0}^\infty$ such that

$$\left(\sum_{n=1}^{\infty}\left(\sum_{m=n^2+1}^{(n+1)^2}\left(|\lambda_m| e^{-n^2} n^{-2m}\right)^{\frac{2p}{2-p}}\right)^{\frac{2-p}{2}}\right)^{\frac{1}{p}} < \infty,$$

if $1 \leq p < 2$.

The space of multipliers $(H_v^\infty(\mathbb{C}), \ell^p)$ continued

Corollary continued.

Let $v(r) = e^{-r}$, $r \in (0, \infty)$ and $1 \leq p \leq \infty$. Then, the space of multipliers $\left(H_v^\infty(\mathbb{C}), \ell_p\right)$ is the set of sequences $(\lambda_m)_{m=0}^\infty$ such that

$$\left(\sum_{n=1}^{\infty} \left(\max_{n^2 < m \leq (n+1)^2} |\lambda_m| e^{n^2} n^{-2m} \right)^p \right)^{\frac{1}{p}} < \infty,$$

if $2 \leq p < \infty$, and

$$\sup_{n \in \mathbb{N}} \left(\max_{n^2 < m \leq (n+1)^2} |\lambda_m| e^{n^2} n^{-2m} \right) < \infty,$$

if $p = \infty$,

Corollary.

Let $v(r) = \exp(-1/(1-r))$, $r \in [0,1[$ and $1 \le p \le \infty$. Then, the space of multipliers $\big(H_v^\infty(\mathbb{D}), \ell_p\big)$ is the set of sequences $(\lambda_m)_{m=0}^\infty$ such that

$$\Big(\sum_{n=1}^{\infty}\Big(\sum_{m=n^4+1}^{(n+1)^4}\big(|\lambda_m| e^{n^2}\Big(1-\frac{1}{n^2}\Big)^{-m}\big)^{\frac{2p}{2-p}}\Big)^{\frac{2-p}{2}}\Big)^{\frac{1}{p}} < \infty,$$

if $1 \le p < 2$,

$$\Big(\sum_{n=1}^{\infty}\Big(\max_{n^4 < m \le (n+1)^4} |\lambda_m| e^{n^2}\Big(1-\frac{1}{n^2}\Big)^{-m}\Big)^{p}\Big)^{\frac{1}{p}} < \infty,$$

if $2 \le p < \infty$, and

$$\sup_{n\in\mathbb{N}}\Big(\max_{n^4 < m \le (n+1)^4} |\lambda_m| e^{n^2}\Big(1-\frac{1}{n^2}\Big)^{-m}\Big) < \infty,$$

if $p = \infty$,

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